Nuclear response to electromagnetic probes: toward spectroscopically accurate many-body theory

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Outline: The RNFT solution to the nuclear many-body problem

- Nuclear structure theory has strived for decades to achieve accurate computation of nuclear spectra, but this is still difficult

- We address this problem by developing the relativistic nuclear field theory (RNFT) via the model-independent Equation of Motion (EOM) framework in the universal QFT language, quantifying fermionic correlation functions (FCFs)

- The theory (i) is transferable across the energy scales, (ii) capable of identifying the bottlenecks of the existing nuclear structure approaches, and (iii) opens the door to spectroscopic accuracy

- In this formulation, dynamical interaction kernels of the FCF EOMs are the source of the richness of the nuclear wave functions in terms of their configuration complexity and the major ingredient for an accurate description with quantified uncertainties

- Electromagnetic and weak probes are the most informative ones for benchmarking the theory and constitute the majority of applications: selected highlights for nuclear excited states

- Open problems

Exact "ab initio" equation of motion (EOM) for the two-fermion correlation function with an unspecified NN interaction

$$H = \sum\!\!\!\!\!\!\int_{12} \bar\psi_1(-i\gamma\cdot\nabla + M)_{12}\psi_2 + \frac{1}{4}\sum\!\!\!\!\!\!\int_{1234} \bar\psi_1\bar\psi_2\bar v_{1234}\psi_4\psi_3 \;+\; \ldots \;=\; T + V^{(2)} + \ldots$$

Bare (Relativistic) **Hamiltonian**

Particle-hole correlator or two-time propagator (response function):

$$R_{12,1'2'}(t-t') = -i\langle T(\bar\psi_1\psi_2)(t)(\bar\psi_{2'}\psi_{1'})(t')\rangle$$

In-medium scattering amplitude spectra of excitations, decays, …

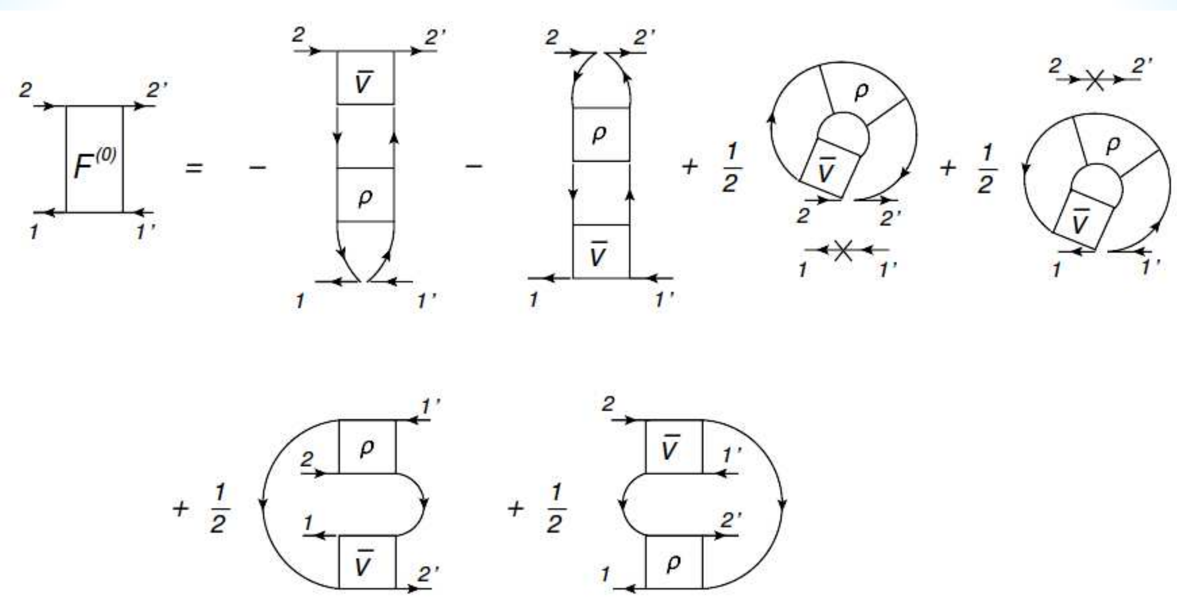
***EOM**: Bethe-Salpeter-(Dyson) Equation (**)*

$$R(\omega) = R^{(0)}(\omega) + R^{(0)}(\omega)F(\omega)R(\omega)$$

Irreducible kernel (exact): in-medium "interaction"

$$F(t-t') = F^{(0)}\delta(t-t') + F^{(r)}(t-t')$$

Instantaneous term ("bosonic" mean field):
Short-range correlations

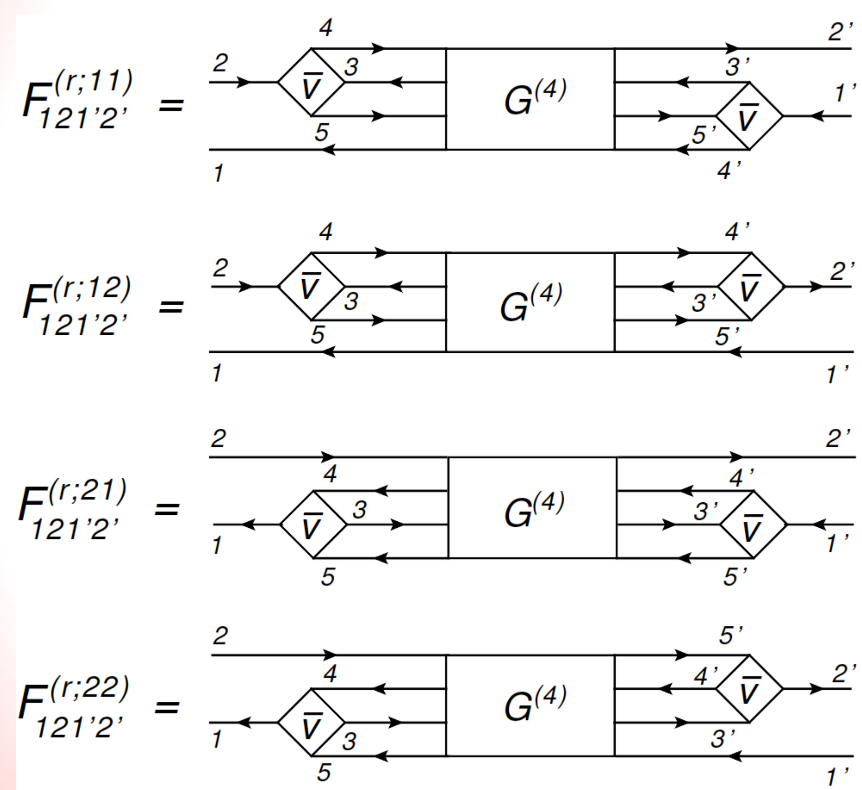


Self-consistent mean field $F^{(0)}$, where

$$\rho_{12,1'2'} = \delta_{22'}\rho_{11'} - i\lim_{t'\to t+0} R_{2'1,21'}(t-t')$$

*contains the full solution of (**) **including the dynamical term!***

t-dependent (dynamical) term:
Long-range correlations:
Couples to higher-rank FCFs



Symmetric form:

P. Schuck, J. Dukelsky, S. Adachi, et al.

Non-Symmetric form: see

J. Schwinger, F. Dyson, et al.

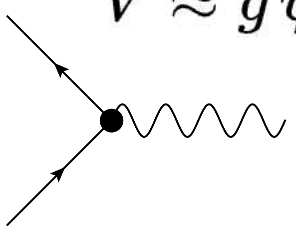
E.L. & P. Schuck, PRC 100, 064320 (2019)

Language remarks

Ab initio…

- *Ab initio* = "from the beginning" (lat.), in science and engineering "from first principles"
- ***First-principle physics theory does not exist***
  Even the Standard Model (SM) is an effective (field) theory:

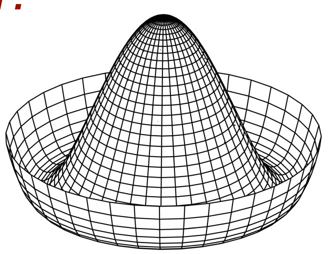
$$V \approx g\bar{\psi}\phi\psi$$



~20 parameters:
masses, coupling constants, etc.

Open questions (besides GR):

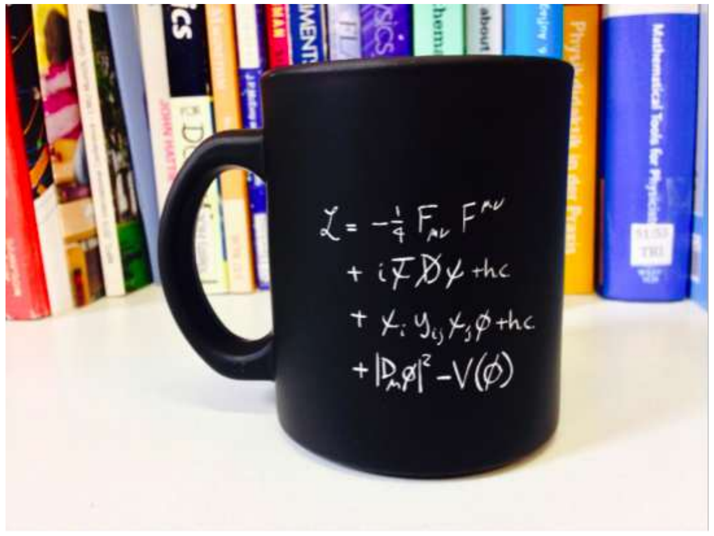
- *Why the SM interactions are as they are?*
- *What is the origin of the Higgs potential?*
- *Dark matter, neutrinos,… ???*



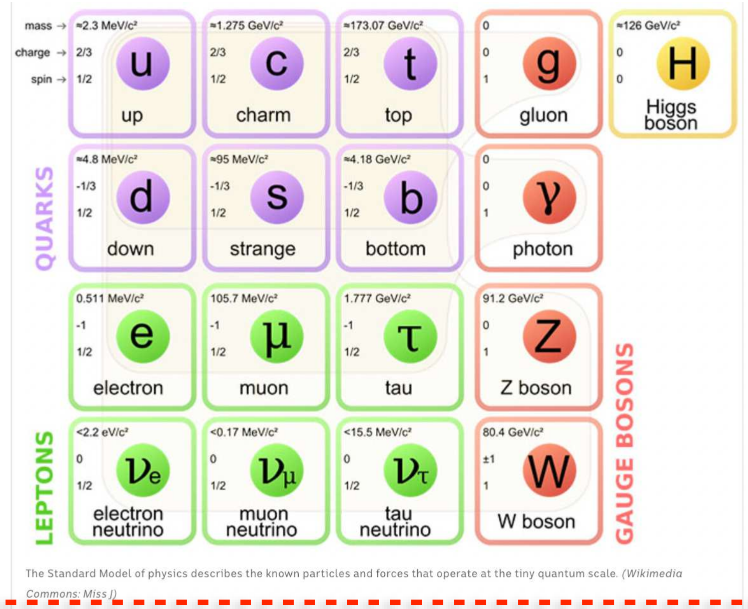
Higgs mechanism of mass generation

$$\mathcal{L} = (\partial_\mu\phi)^\dagger(\partial^\mu\phi) - V(\phi),$$

$$V(\phi) = \mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2.$$



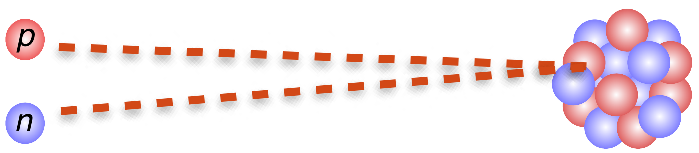
The Standard Model



"Bare" ***Interaction*** *(aka forces, potentials):*

$V(\mathbf{r}_1 - \mathbf{r}_2)$

Ab initio…

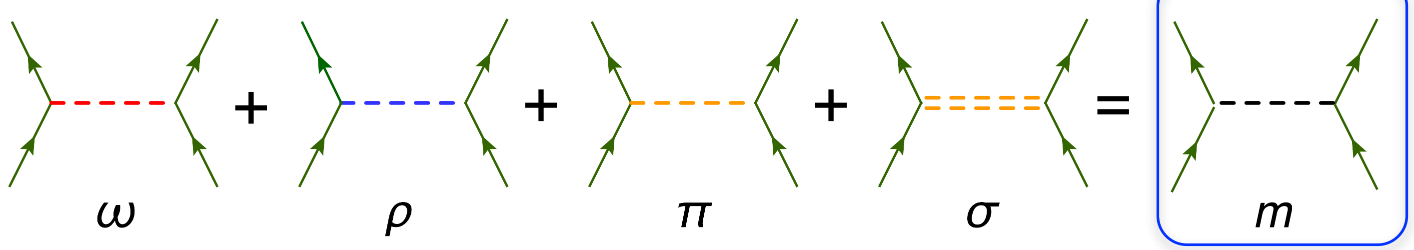


- *In* ***Theoretical Physics****: knowing the bare (vacuum) interaction between two particles, quantitatively describe the many-particle system*
- *"Ab initio = UV-complete" as opposed to "effective"*
- *Interactions adjusted to many-particle systems (e.g., finite nuclei) are not ab initio but effective*

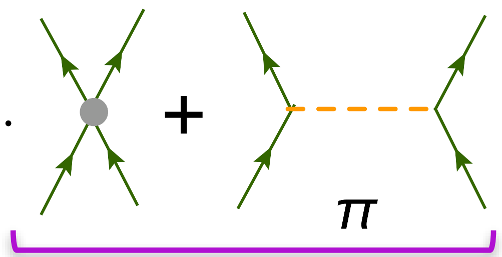
Linking the nucleon-nucleon (NN) interaction and the many-body theory

Quantum Hadrodynamics (QHD)

Leading order:



cf.

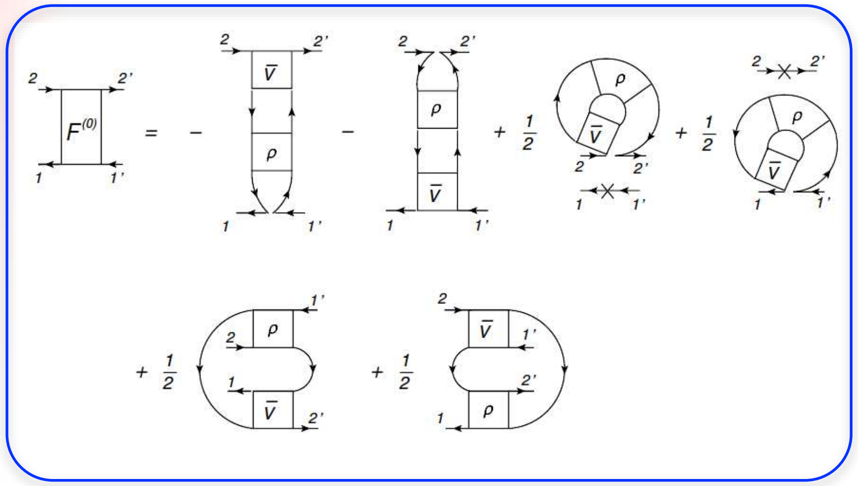


RNFT (this work): non-PT, in-medium, effective couplings

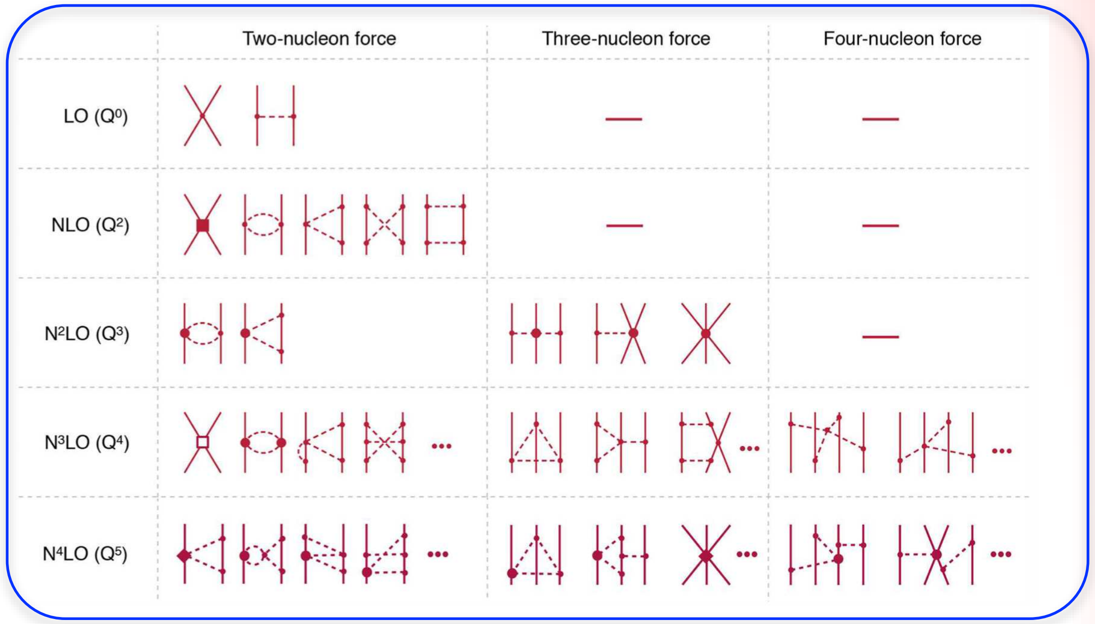
Beyond the leading order:

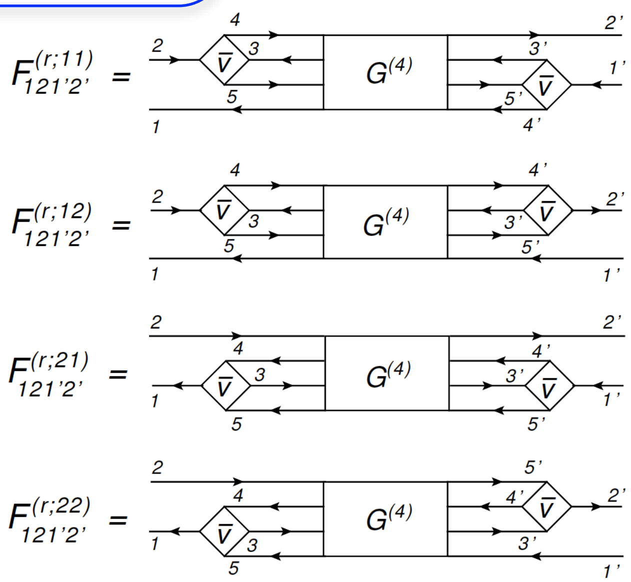
cf. χEFT: PT in the vacuum

E. Epelbaum et al., Front. Phys. 8, 98 (2020)



Static





+ Dynamical

+ Dynamical

Many-body… "solver" (?)

- *Standard "solvers" are too simplistic*
- *Bare NN and many-body theory should be linked consistently*
- ***In the medium, the power counting changes, etc.***

The in-medium power counting is associated with emergent scales in large systems

Dynamical kernels: bridging the scales

- ***Quantum many-body problem in a nutshell:*** *Direct EOM for $G^{(n)}$ generates $G^{(n+2)}$ in the (symmetric) dynamical kernels and further high-rank correlation functions (CFs); an equivalent of the BBGKY hierarchy or Schwinger-Dyson equations. $N_{Equations} \sim N_{Particles}$ & Coupled 🙈 !!!*
- ***Non-perturbative solutions:***

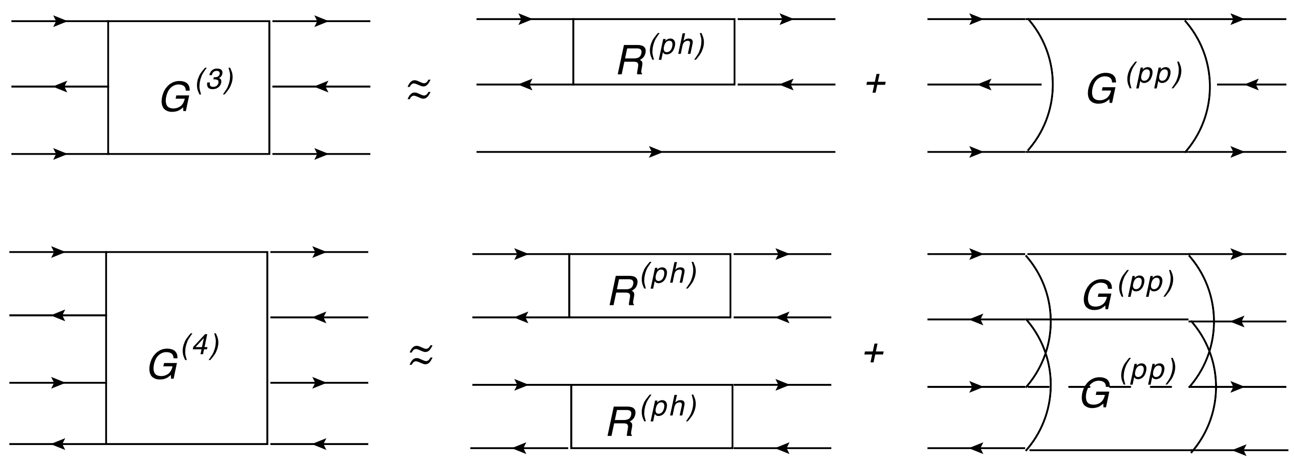
Cluster decomposition [QFT, S. Weinberg]

$$✦\ G^{(3)} = G^{(1)} G^{(1)} G^{(1)} + G^{(2)} G^{(1)} + \Xi^{(3)}$$

$$✦\ G^{(4)} = G^{(1)} G^{(1)} G^{(1)} G^{(1)} + G^{(2)} G^{(2)} + G^{(3)} G^{(1)} + \Xi^{(4)}$$

Leading at: ***weak*** (*self-consistent GFs, second RPA, shell-model etc.*) ***intermediate*** (*phonon coupling* **this work**) ***strong*** (**Faddeev + future work**) *coupling*

Beyond weak coupling:

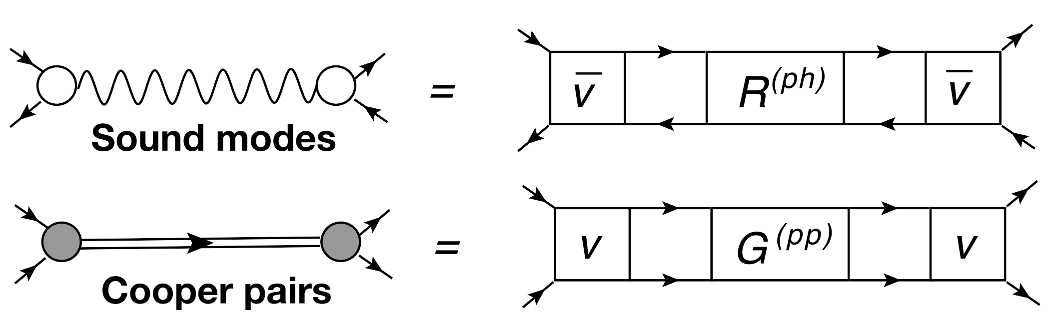


- *P. C. Martin and J. S. Schwinger, Phys. Rev.115, 1342 (1959).*
- *N. Vinh Mau, Trieste Lectures 1069, 931 (1970)*
- *P. Danielewicz and P. Schuck, Nucl. Phys. A567, 78 (1994)*
- *…*

Exact mapping: particle-hole (2q) quasibound states

Emergence of effective "bosons" (phonons, vibrations):

Emergence of superfluidity:



Quasiparticle-vibration coupling (qPVC) in nuclei. Cf. NFT

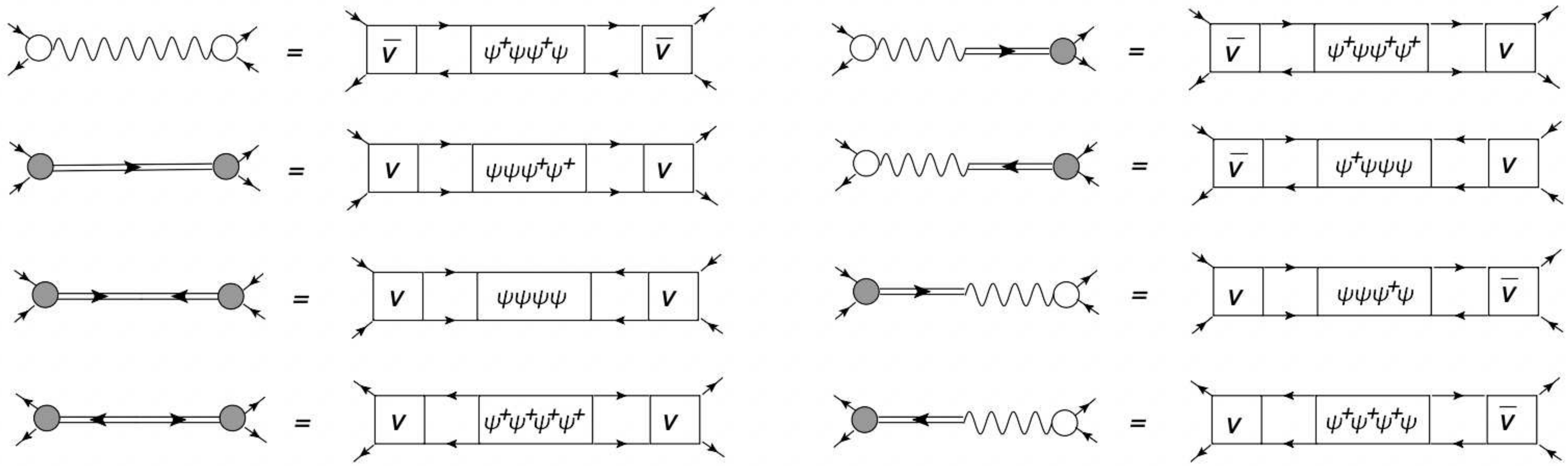
Diquarks in hadrons

Cf. C. Popovici, P. Watson, and H. Reinhardt [PRD83, 025013 (2011)]

“Ab-initio” qPVC in superfluid systems

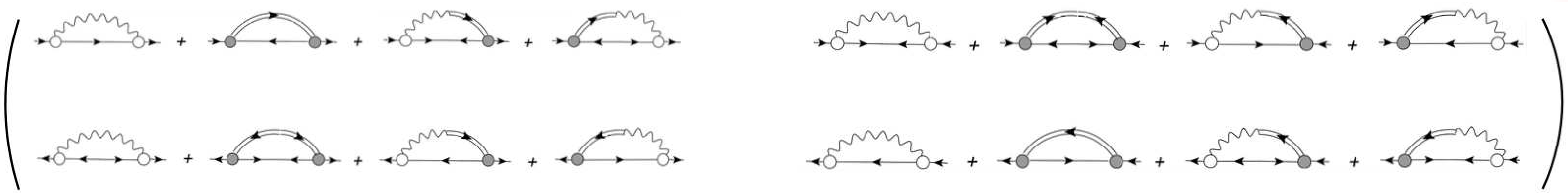
Superfluid dynamical kernel: adding particle-number violating contributions

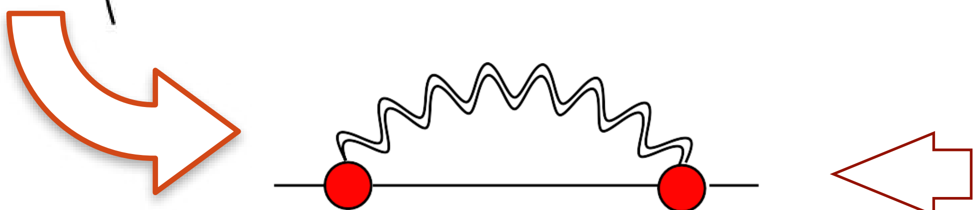
Mapping on the qPVC in the canonical basis



Quasiparticle dynamical self-energy (matrix): normal and pairing phonons are unified

$\hat{\Sigma}^r =$





$\mathcal{W}$

Bogoliubov transformation

Compact form, (almost) as simple as non-superfluid

Cf.: Quasiparticle static self-energy (matrix) in HFB

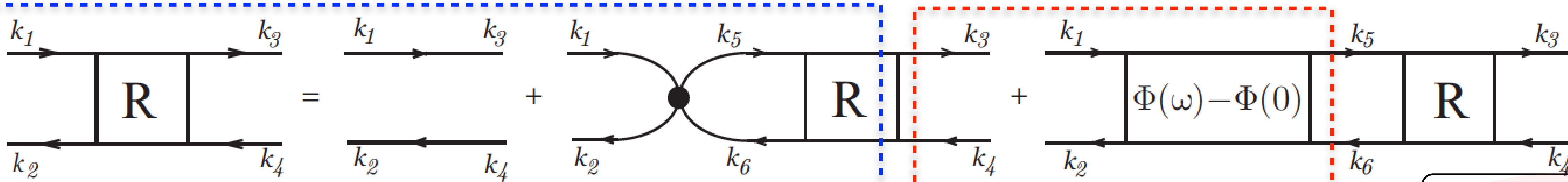
$$\hat{\Sigma}^0 = \begin{pmatrix} \tilde{\Sigma}_{11'} & \Delta_{11'} \\ -\Delta^*_{11'} & -\tilde{\Sigma}^T_{11'} \end{pmatrix}$$

E.L., Y. Zhang, PRC 104, 044303 (2021)
Y. Zhang et al., PRC 105, 044326 (2022)

Recently formulated in the HFB basis keeping 4x4 block matrix structure at zero and finite temperature: S. Bhattacharjee, E.L., arXiv:2412.20751

Nuclear response: toward a complete theory

toward ab initio <= static kernel V *dynamic kernel => toward more complex configurations*



Dyson-Bethe-Salpeter Equation:

$$R(\omega) = R^0(\omega) + R^0(\omega)\,[V + \Phi(\omega) - \Phi(0)\,]\,R(\omega)$$

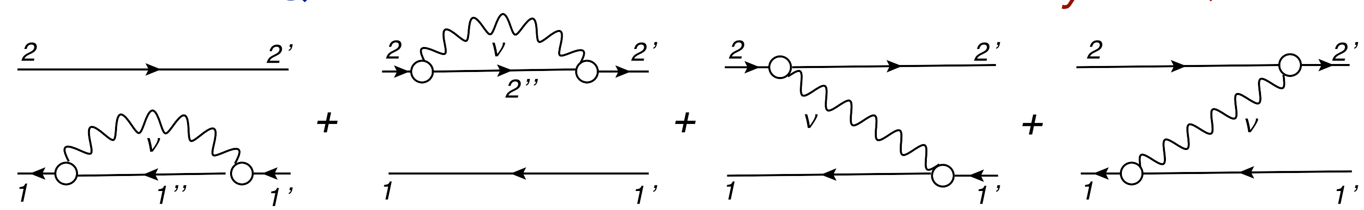
Subtraction [V.Tselyaev 2013]

for effective interactions V [P. Ring, A.Afanasjev, et al: CEDF]

QRPA *Beyond QRPA*

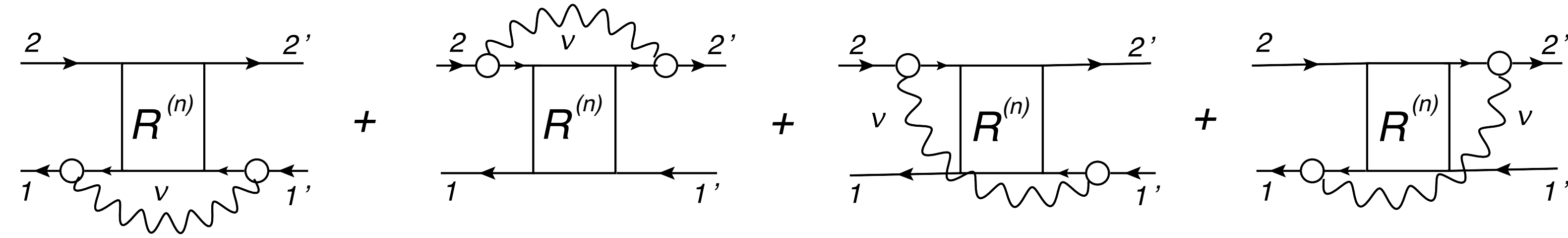
NFT is reproduced as the leading approximation

$\Phi_{21,2'1'} =$



Extended NFT: closed cycle

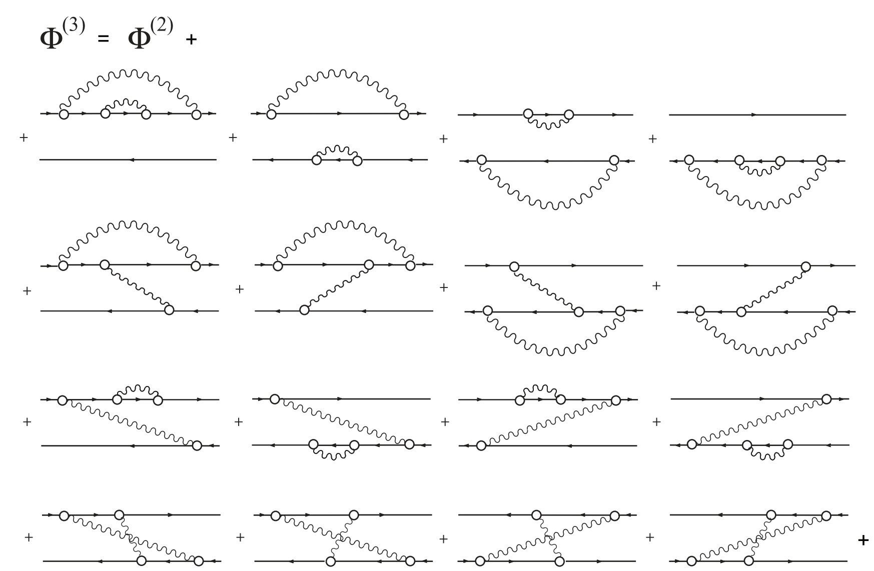
$\Phi^{(n+1)}_{212'1'} =$



Generalized approach for the correlated propagators

n-th order: E.L. PRC 91, 034332 (2015)

Ab-initio formulation,
Φ(3) implementation; 2q+2phonon correlations:
E.L., P. Schuck, PRC 100, 064320 (2019)



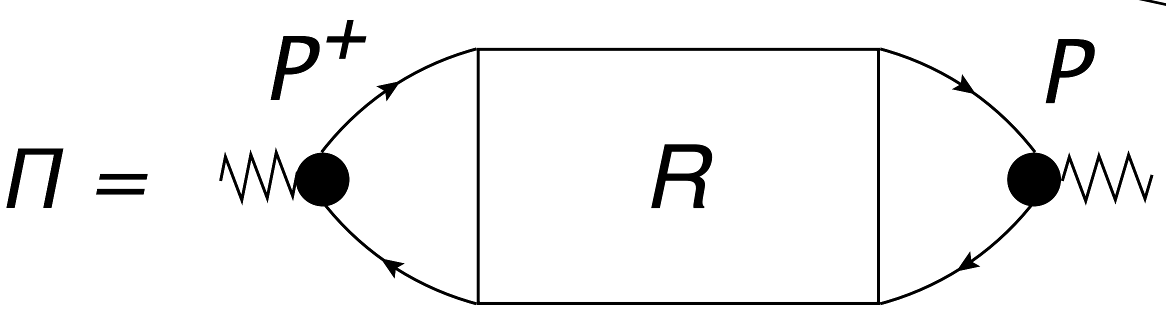
2q+2phonon configurations in the qPVC expansion:

Included in all orders (non-perturbatively)

Response to an external field: strength function

Nuclear Polarizability: $$\Pi_{PP}(\omega) = P^{\dagger}R(\omega)P := \sum_{k_1k_2k_3k_4} P^*_{k_1k_2} R_{k_1k_4,k_2k_3}(\omega) P_{k_3k_4}$$

External field



$$e\sum_{i=1}^{Z} r_i^2$$ *Monopole*

Dipole (CM corrected)

$$\frac{eN}{A}\sum_{i=1}^{Z} r_i Y_{1M}(\hat{\mathbf{r}}_i) - \frac{eZ}{A}\sum_{i=1}^{N} r_i Y_{1M}(\hat{\mathbf{r}}_i)$$

$$e\sum_{i=1}^{Z} {r_i}^L Y_{LM}(\hat{\mathbf{r}}_i) \qquad L \geq 2,$$ *Multipole*

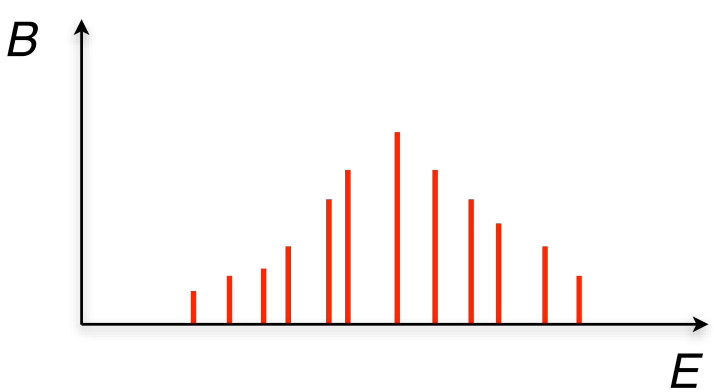
Strength function:

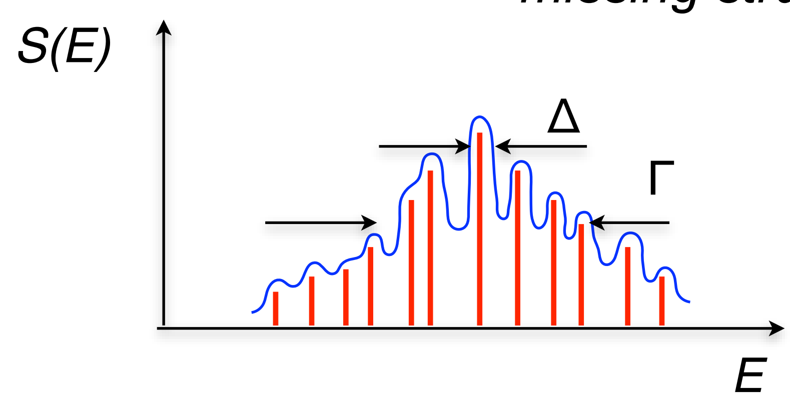
$$S(E) = \sum_n B_n \delta(E - E_n) = -\frac{1}{\pi}\lim_{\Delta\to 0} \Im \Pi_{PP}(E + i\Delta)$$

Transition probability:

$$B_n = |\langle n|P|0\rangle|^2$$

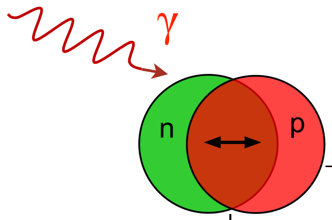
Artificial width Δ (continuum if discretized, missing structure): a parameter



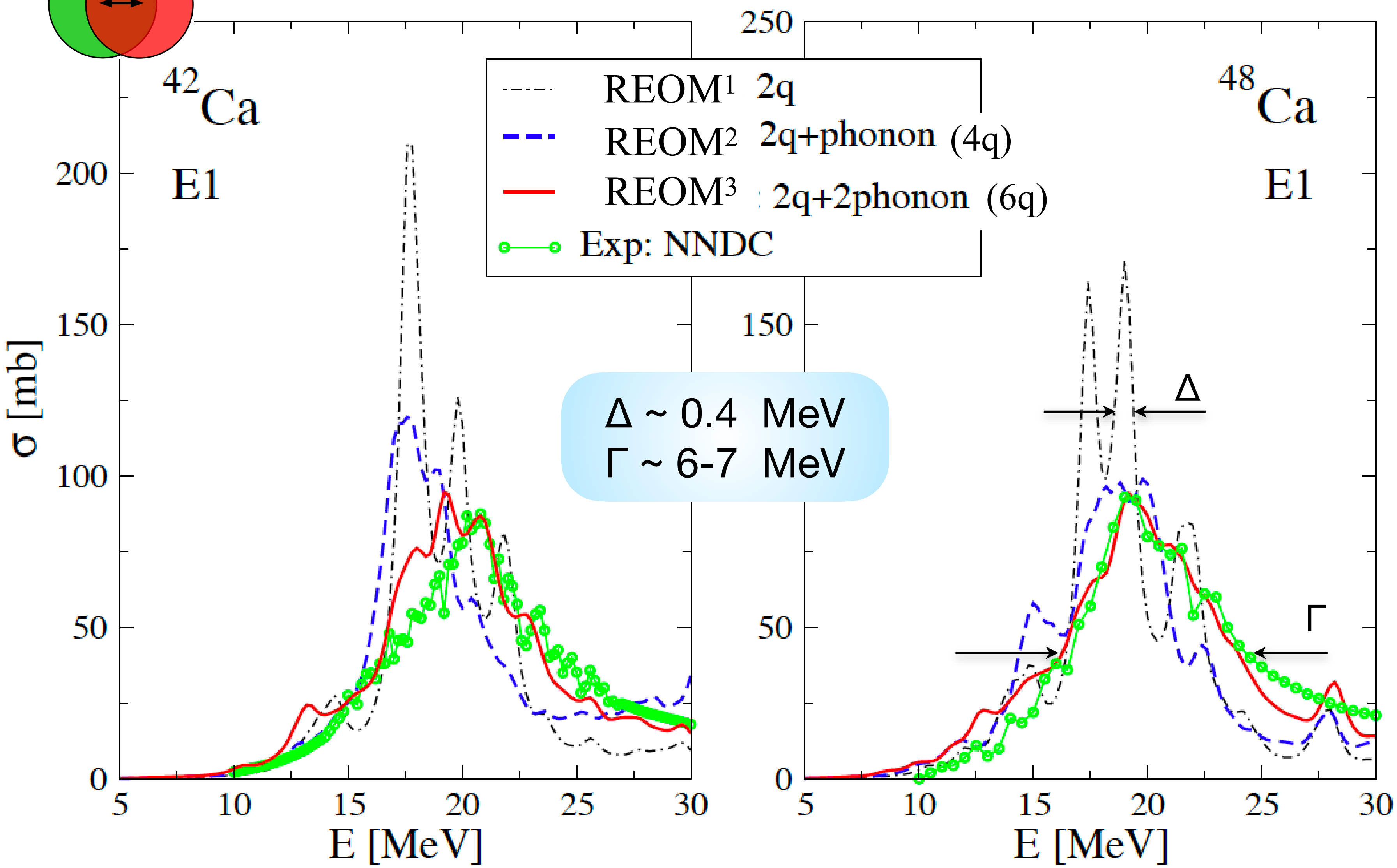


Spreading width Γ (many-body physics): the result of calculations

Toward spectroscopically accurate theory: REOM³/RQTBA³



Giant Dipole Resonance (GDR) in Ca isotopes



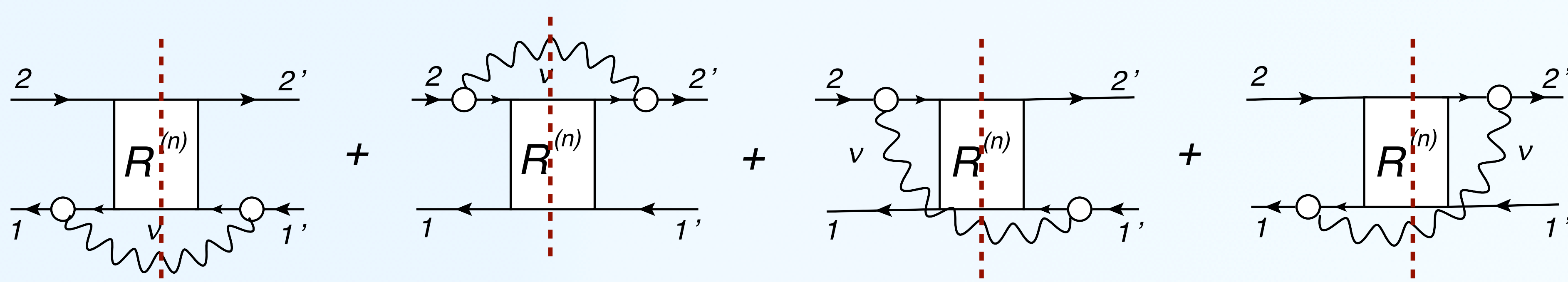
- *On each iteration, the complex configurations enforce fragmentation and spreading toward higher and lower energies*
- *Exp. Data: V.A. Erokhova et al., Bull. Rus. Acad. Phys. 67, 1636 (2003)*
- *REOM³ demonstrates an overall systematic improvement of the description of nuclear excited states as compared to REOM² in a broad energy range*

E.L., P. Schuck, PRC 100, 064320 (2019)

Continuing the hierarchy of the dynamical kernels:

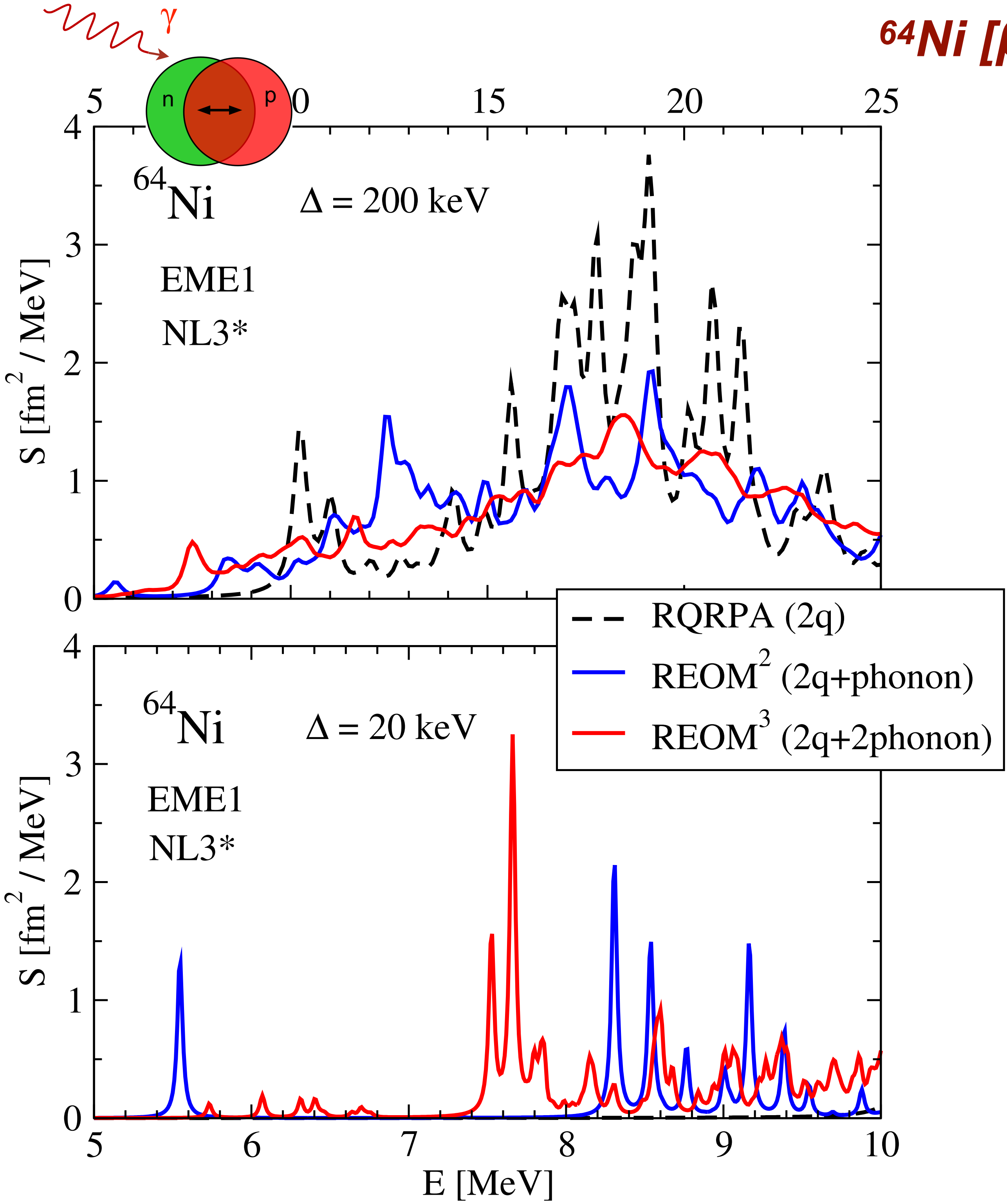
n = 0 (no dynamical)
n = 1 (2q⊗phonon)
n = 2 (2q⊗2phonon)

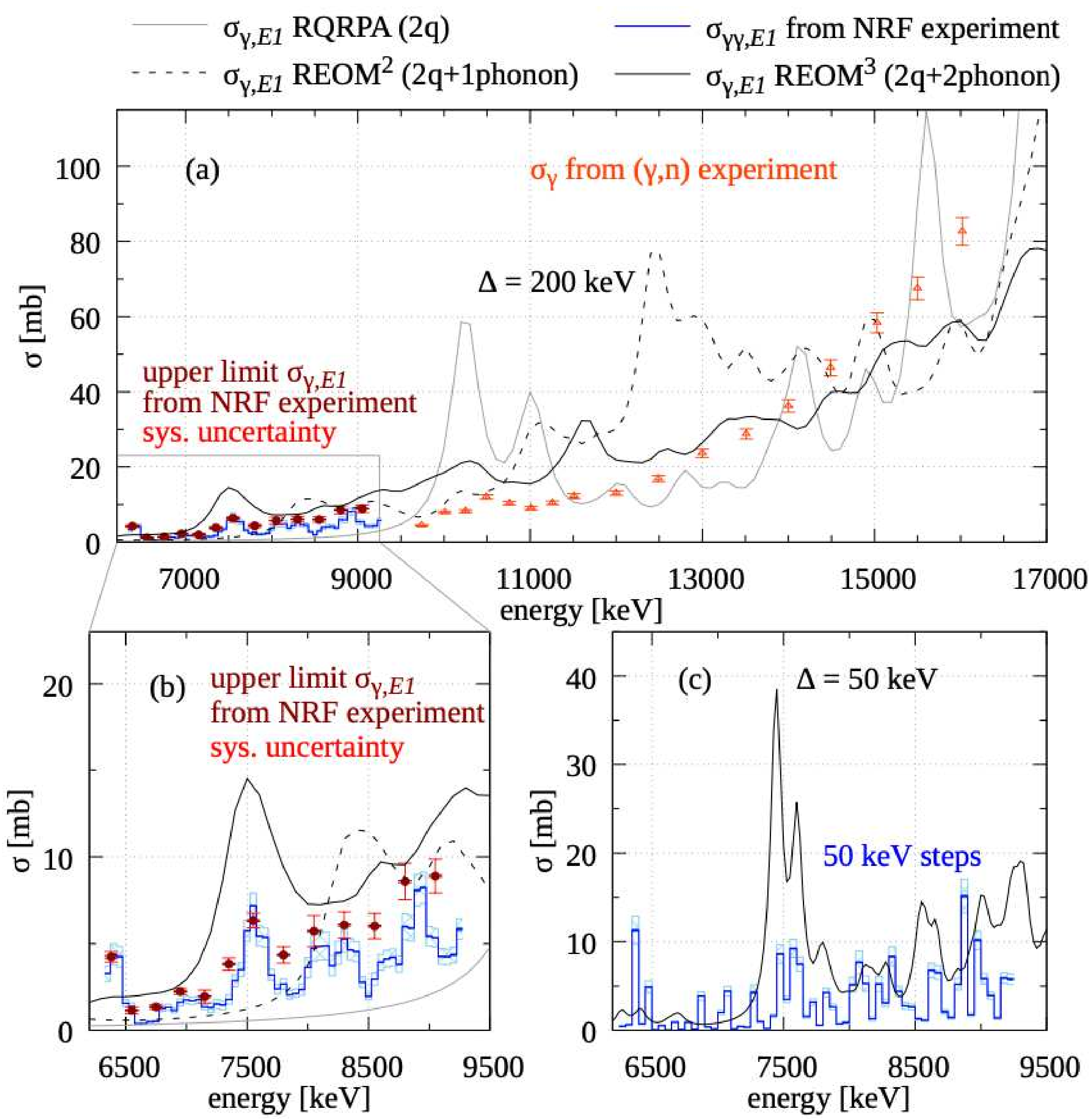
Each elementary 2q mode produces a multitude of states via fragmentation that repeats on the 4q level and so on. Cf.: Fractal self-similarity (nesting)



Transitional and moderately deformed systems: ^{64}Ni

^{64}Ni [$\beta_2 \sim 0.16$]:





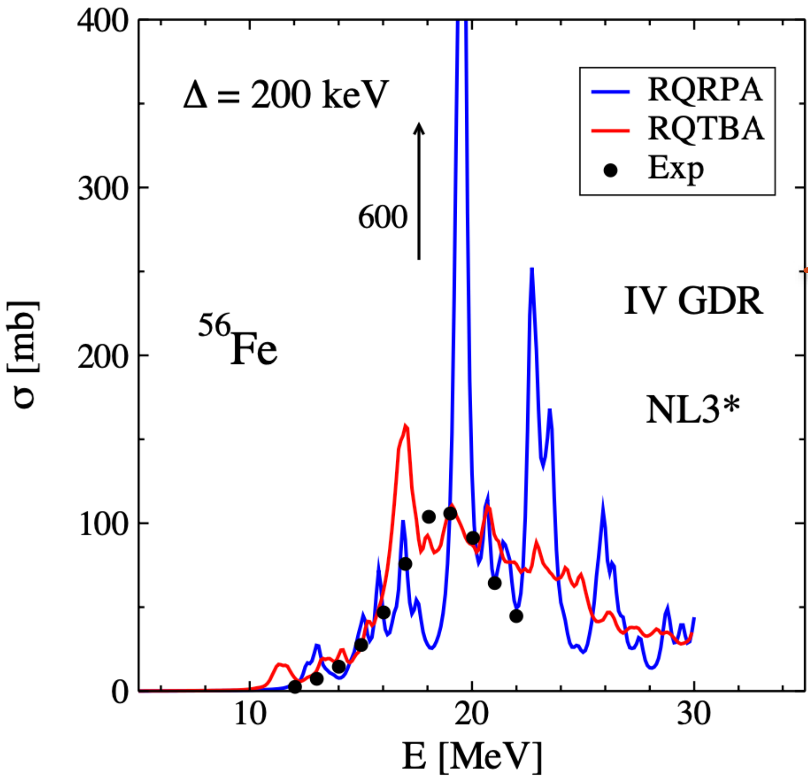
M. Müscher, A. Zilges, E.L. et al.,
PRC109, 044318 (2024)

Spherical bases are used: correlations
“generate” non-trivial geometry

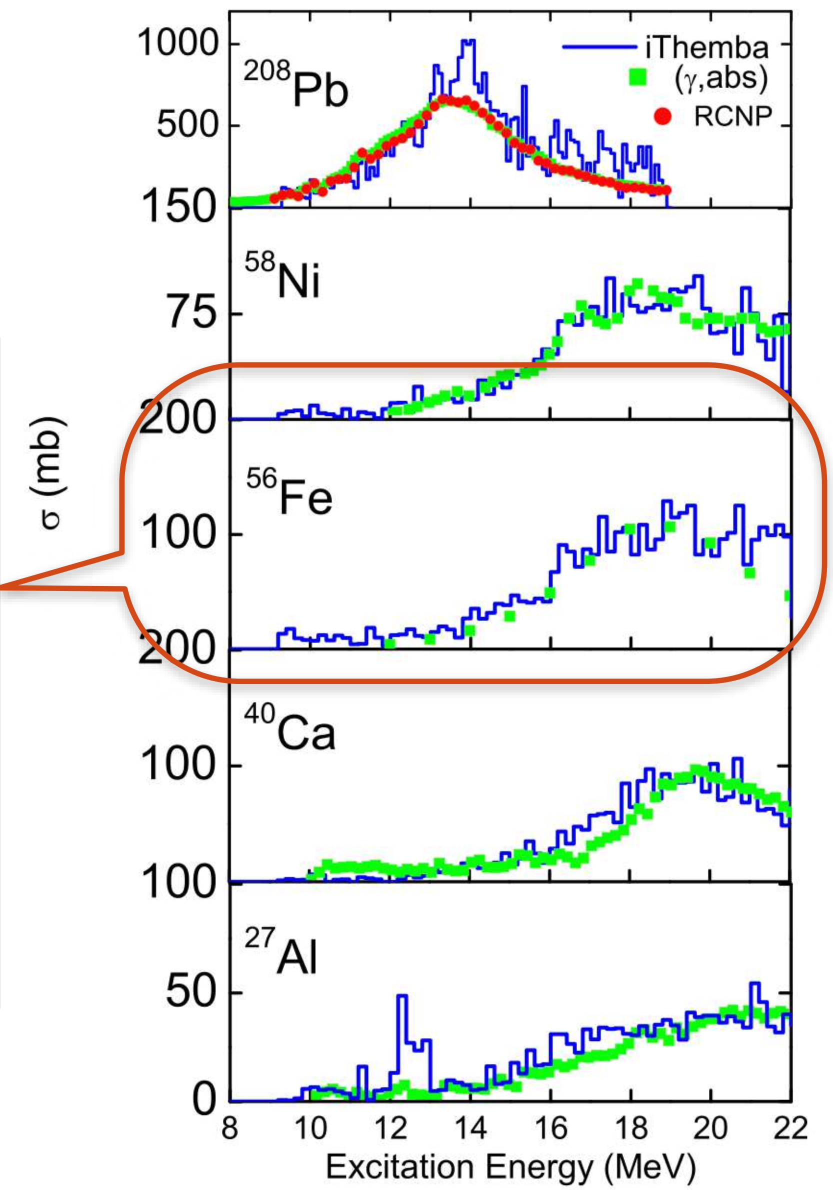
The IVGDR in ^{56}Fe: REOM[1,2] and ^{50}Ti: REOM[1-3]

^{56}Fe [$\beta_2 \sim 0.25$]:

Quadrupole instabilities in spherical QRPA



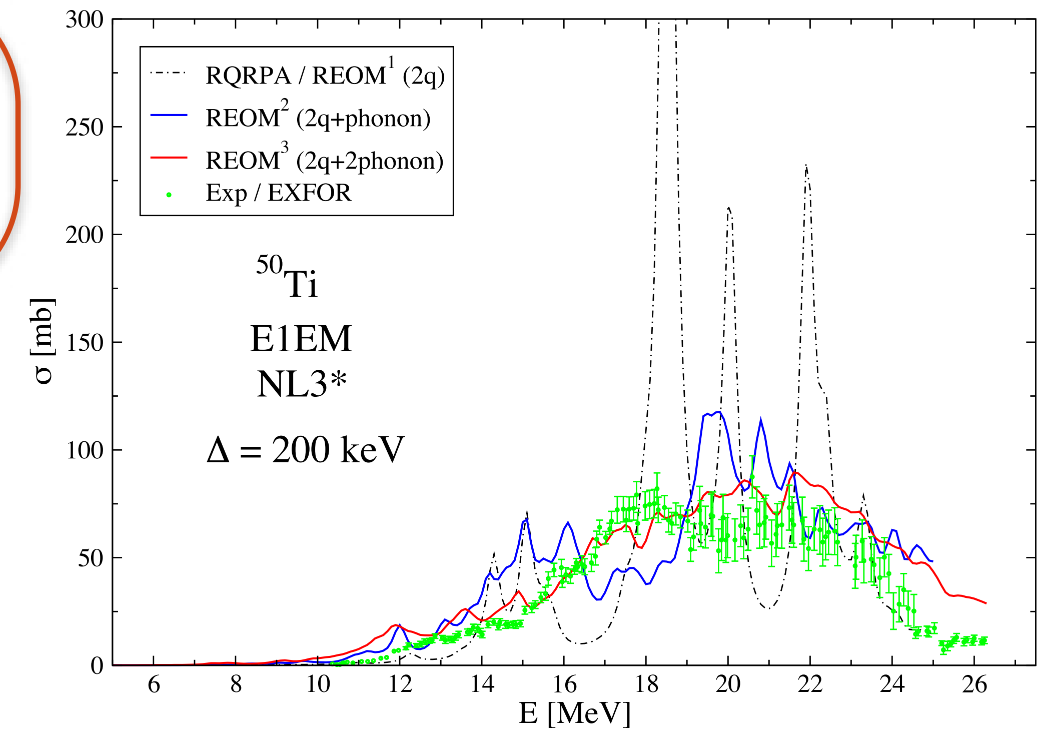
Data: S.S. Borodina, et al., Phys. Rep. 2000, 6/610 (2000). [scan from M. Jingo et al.]



*M. Jingo et al., Eur. Phys. J. A (2018) **54**: 234*

^{50}Ti [$\beta_2 \sim 0.17$]:

No quadrupole instabilities



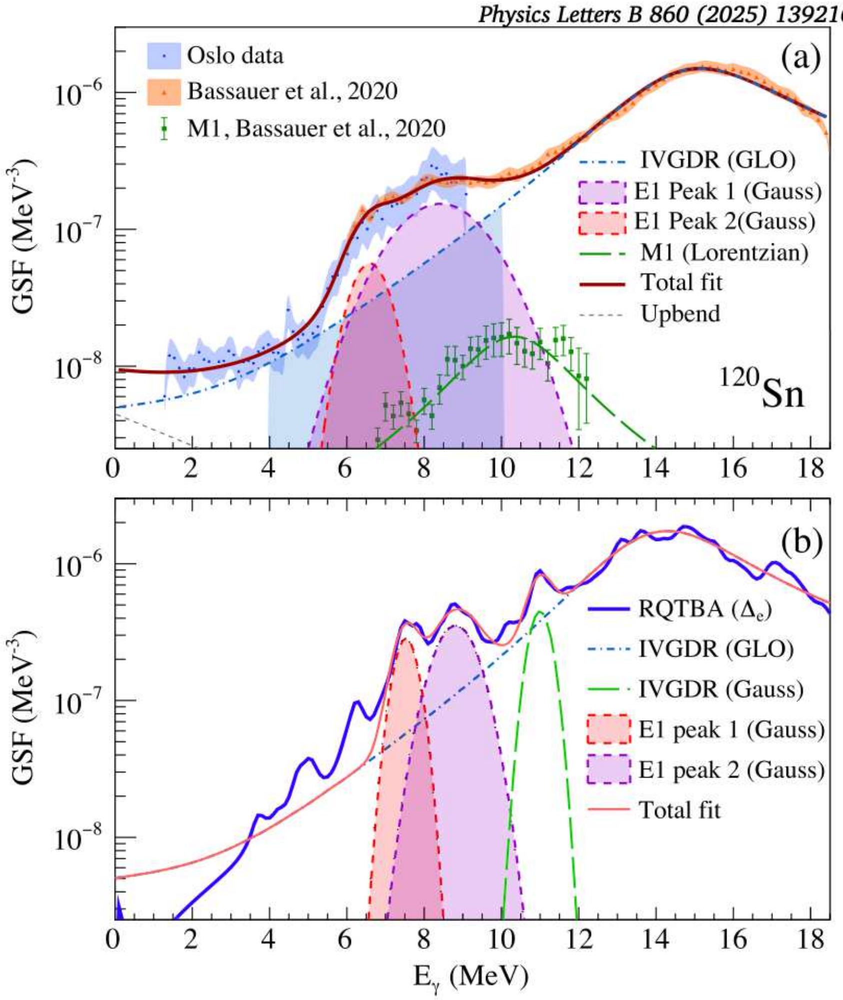
Data: NNCD/EXFOR

- *Fully self-consistent (only the universal CDFT NL3* parameters (8) are used, plus pairing strength)*
- *But: Some uncertainty on the pairing interaction (currently addressed)*
- *Tractable in spherical basis (except for large β_2 when "unstable" 2^+_1 state is problematic)*
- *Will be compared to the axially deformed calculations*
- *More cases are in progress*

Pygmy dipole resonance (PDR): structural properties on the REOM^2 level

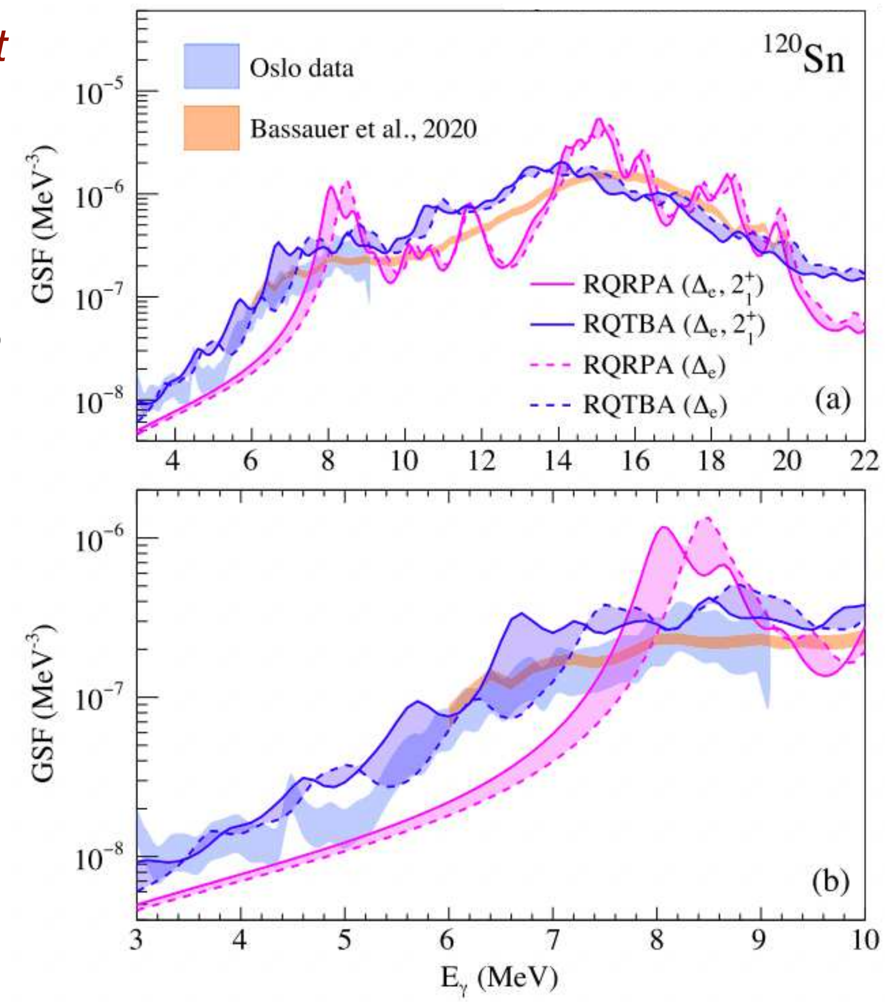
GDR and PDR in the tin chain: $^{112-120}$Sn

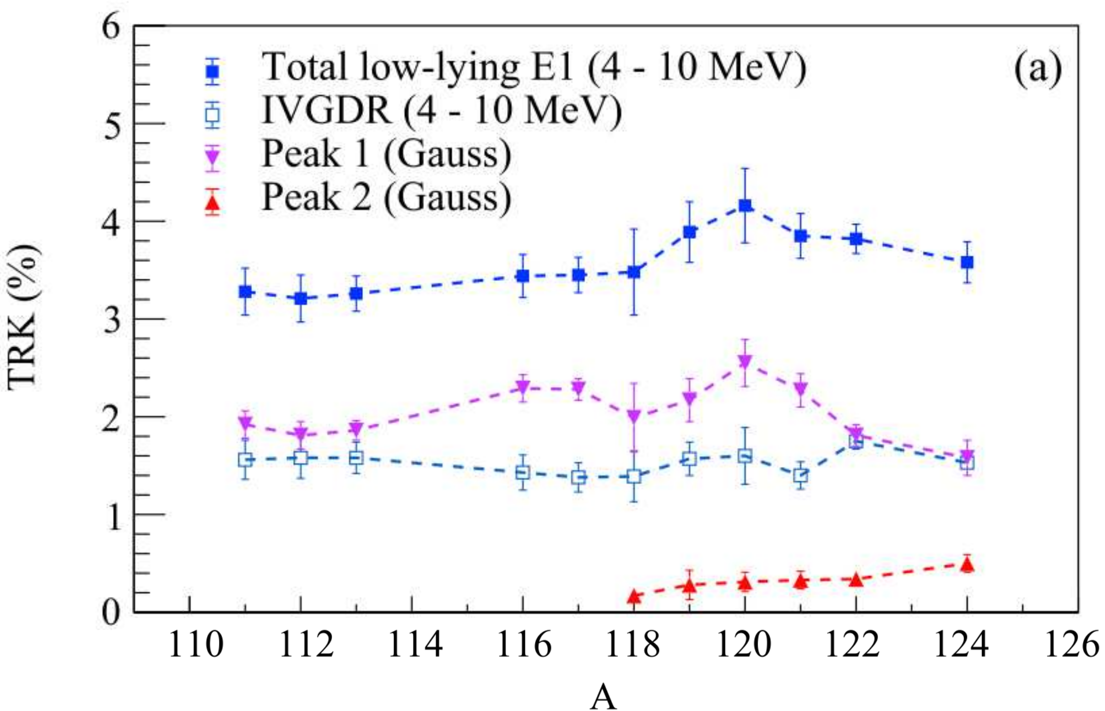
Physics Letters B 860 (2025) 139216



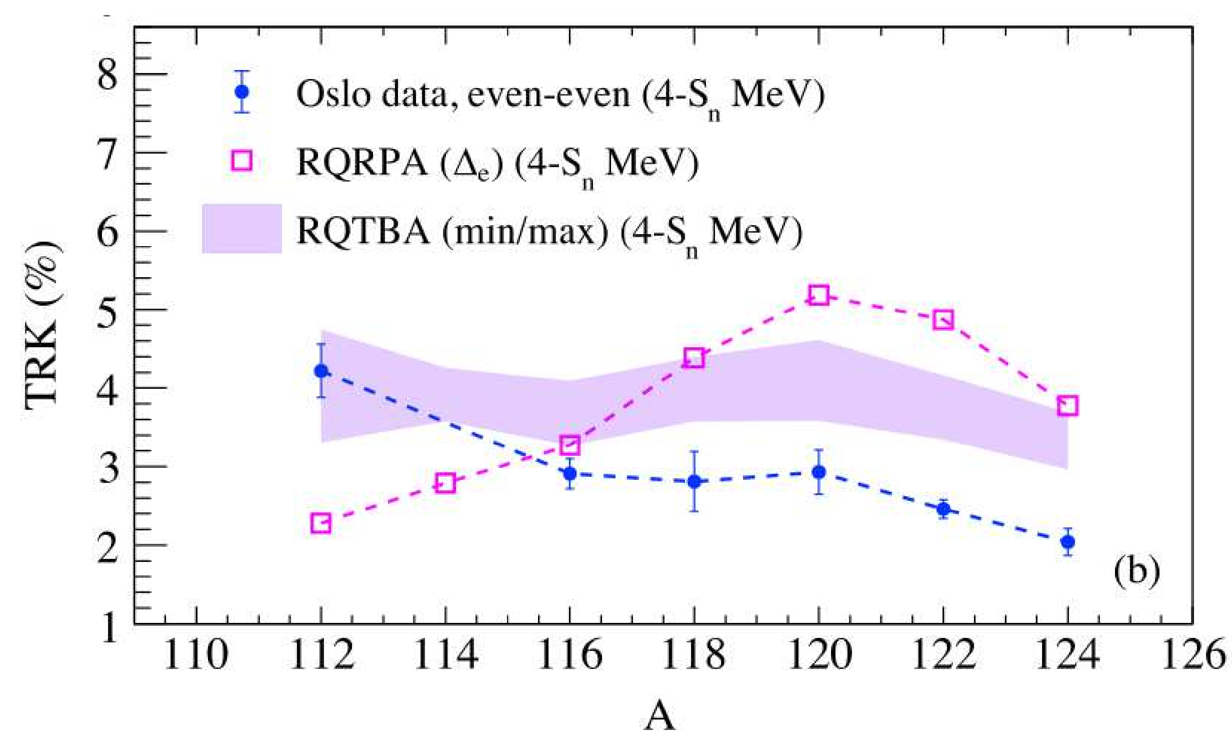
- Data analysis indicate the two-component structure of the low-energy dipole strength
- RQRPA does not describe the data well but remains a good reference point.
- 2q+phonon/RQTBA/REOM^2 is sensitive to the details of the superfluid pairing, particularly the pairing gap, especially at low energies.
- Pairing is the anomalous part of the mean field and also poorly constrained.
- Nevertheless, we could document the formation of the two-peak structure and reasonable total strength below S_n with theoretical uncertainties.

M. Markova, P. von Neumann-Cosel, E. L., PLB 860, 139216 (2025)

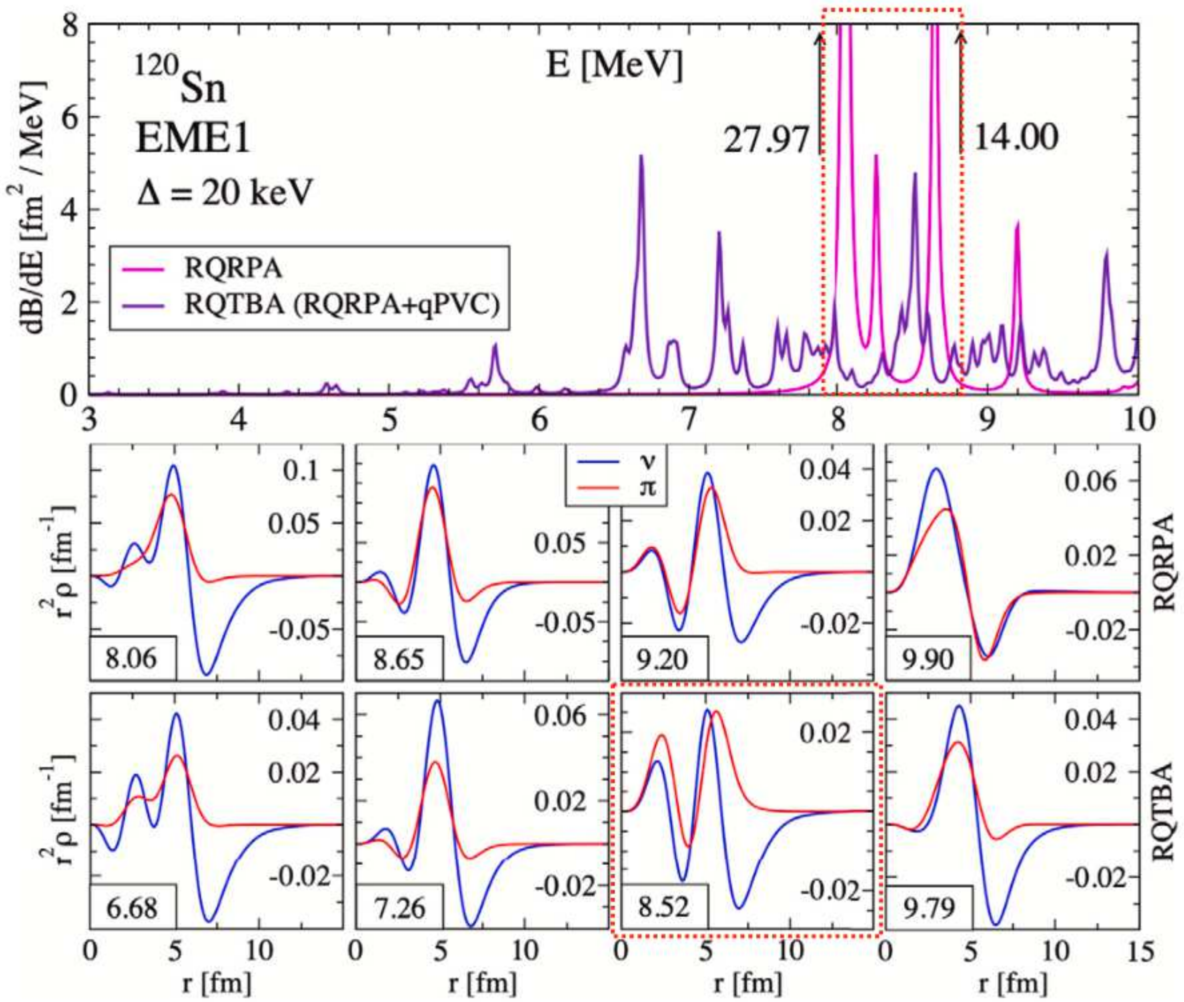


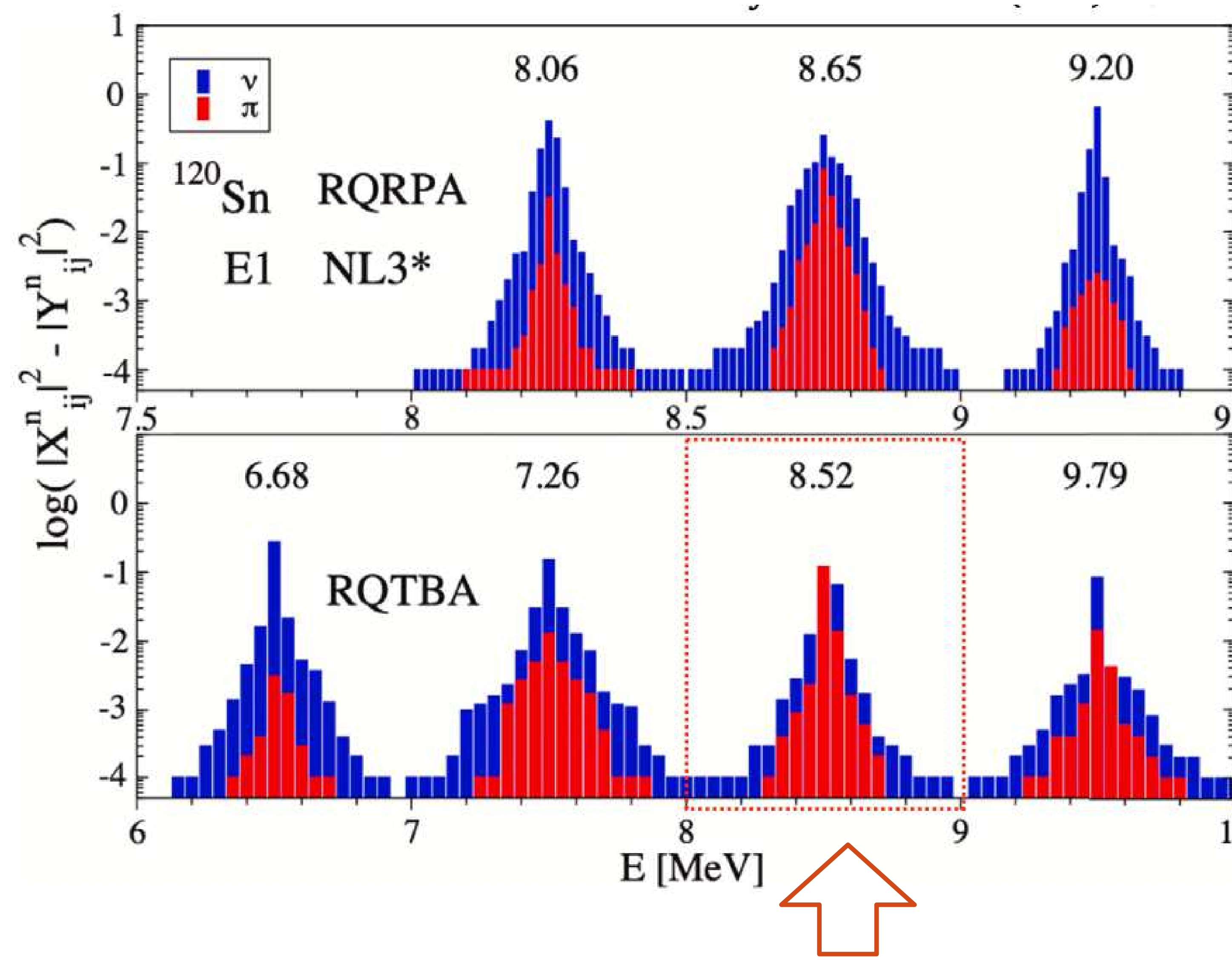


Sum rule fractions in PDR



Low-energy dipole strength (LEDS): structural properties





Gamma transitions between excited states:

Dominate the upper component of LEDS

$$\mathcal{F}_{mn} = \langle m|\mathcal{F}|n\rangle = f_{ij}(\mathcal{X}^{m*}_{ik}\mathcal{X}^{n}_{jk} + \mathcal{Y}^{m*}_{jk}\mathcal{Y}^{n}_{ik})$$

$$\mathcal{X}^{n}_{jk} = \langle 0|\alpha_k\alpha_j|n\rangle \qquad \mathcal{Y}^{n}_{jk} = \langle 0|\alpha^\dagger_j\alpha^\dagger_k|n\rangle$$

$$F_{LM} = e\sum_{i=1}^{Z} r_i{}^L Y_{LM}(\hat{\mathbf{r}}_i), \qquad L \geq 2$$

$$F_{1M} = \frac{eN}{A}\sum_{i=1}^{Z} r_i Y_{1M}(\hat{\mathbf{r}}_i) - \frac{eZ}{A}\sum_{i=1}^{N} r_i Y_{1M}(\hat{\mathbf{r}}_i)$$

Couples exclusively to protons

M. Markova, P. von Neumann-Cosel, E. L., PLB 860, 139216 (2025)

Nuclei at the limits of existence: Finite-temperature response with the ph+phonon dynamical kernel

$$R_{12,1'2'}(t-t') = -i\langle \mathcal{T}(\bar\psi_1\psi_2)(t)(\bar\psi_{2'}\psi_{1'})(t')\rangle \to -i\langle \mathcal{T}(\bar\psi_1\psi_2)(t)(\bar\psi_{2'}\psi_{1'})(t')\rangle_T$$

$$< ... > \equiv < 0|...|0 > \quad \to \quad < ... >_T \equiv \sum_n exp\Big(\frac{\Omega - E_n - \mu N}{T}\Big) < n|...|n >$$

averages → *thermal averages*

Method: EOM for Matsubara Green's functions

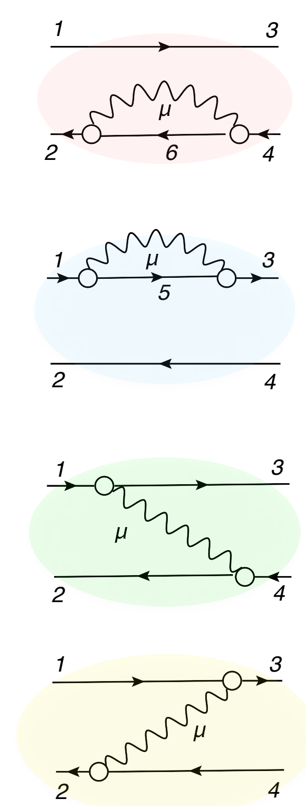
E.L., H. Wibowo, PRL 121, 082501 (2018)
H. Wibowo, E.L., PRC 100, 024307 (2019)

$$\mathcal{R}_{14,23}(\omega,T) = \tilde{\mathcal{R}}^0_{14,23}(\omega,T) + \\ + \sum_{1'2'3'4'} \tilde{\mathcal{R}}^0_{12',21'}(\omega,T)\big[\tilde V_{1'4',2'3'}(T) + \delta\Phi_{1'4',2'3'}(\omega,T)\big]\mathcal{R}_{3'4,4'3}(\omega,T)$$

$$\delta\Phi_{1'4',2'3'}(\omega,T) = \Phi_{1'4',2'3'}(\omega,T) - \Phi_{1'4',2'3'}(0,T)$$

T > 0: *Leading-order 1p1h+phonon dynamical kernel:* *T = 0:*

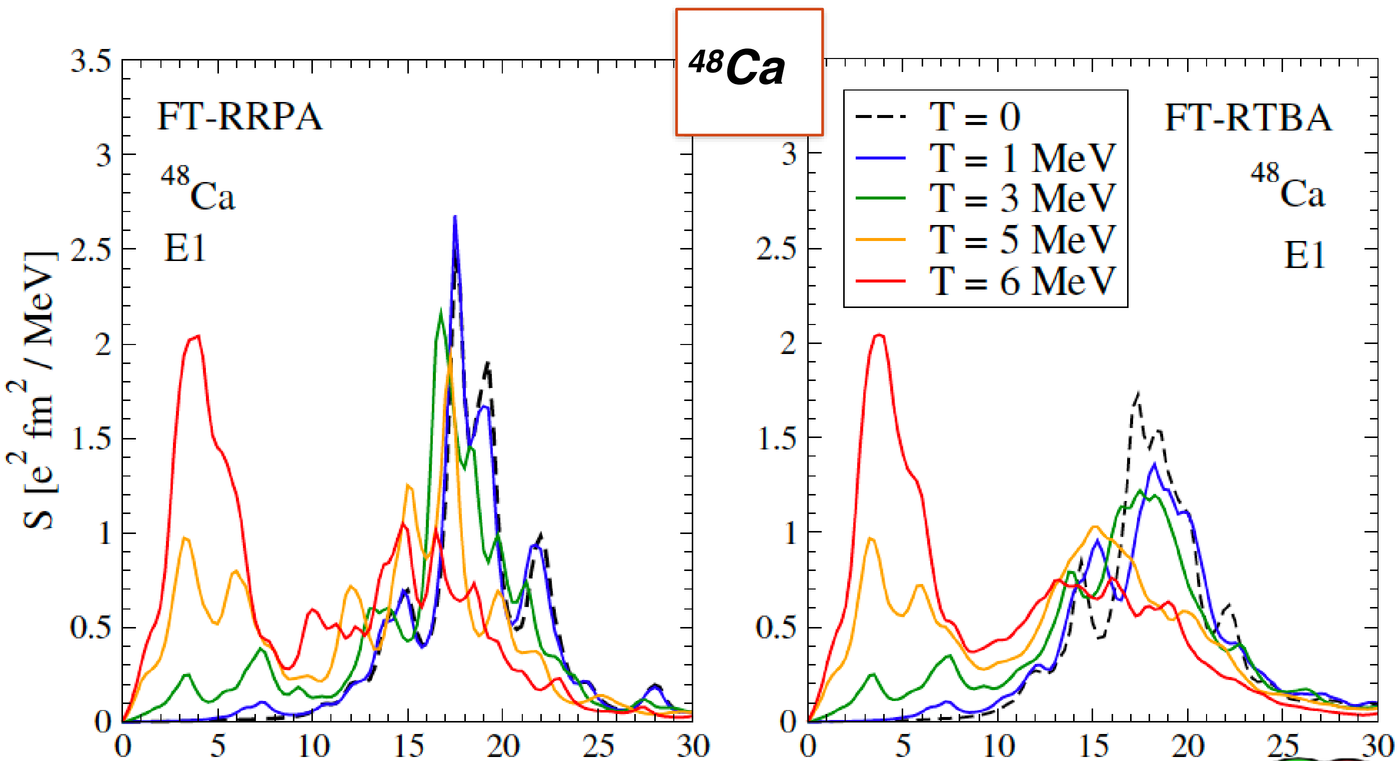
$$\Phi^{(ph)}_{14,23}(\omega,T) = \frac{1}{n_{43}(T)}\sum_{\mu,\eta_\mu=\pm1}\eta_\mu\Big[\delta_{13}\sum_6\gamma^{\eta_\mu}_{\mu;62}\gamma^{\eta_\mu *}_{\mu;64}\times \\ \times\frac{\big(N(\eta_\mu\Omega_\mu)+n_6(T)\big)\big(n(\varepsilon_6-\eta_\mu\Omega_\mu,T)-n_1(T)\big)}{\omega-\varepsilon_1+\varepsilon_6-\eta_\mu\Omega_\mu} + \\ + \delta_{24}\sum_5\gamma^{\eta_\mu}_{\mu;15}\gamma^{\eta_\mu *}_{\mu;35}\times \\ \times\frac{\big(N(\eta_\mu\Omega_\mu)+n_2(T)\big)\big(n(\varepsilon_2-\eta_\mu\Omega_\mu,T)-n_5(T)\big)}{\omega-\varepsilon_5+\varepsilon_2-\eta_\mu\Omega_\mu} - \\ - \gamma^{\eta_\mu}_{\mu;13}\gamma^{\eta_\mu *}_{\mu;24}\times \\ \times\frac{\big(N(\eta_\mu\Omega_\mu)+n_2(T)\big)\big(n(\varepsilon_2-\eta_\mu\Omega_\mu,T)-n_3(T)\big)}{\omega-\varepsilon_3+\varepsilon_2-\eta_\mu\Omega_\mu} - \\ - \gamma^{\eta_\mu *}_{\mu;31}\gamma^{\eta_\mu}_{\mu;42}\times \\ \times\frac{\big(N(\eta_\mu\Omega_\mu)+n_4(T)\big)\big(n(\varepsilon_4-\eta_\mu\Omega_\mu,T)-n_1(T)\big)}{\omega-\varepsilon_1+\varepsilon_4-\eta_\mu\Omega_\mu}\Big],$$

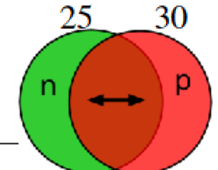


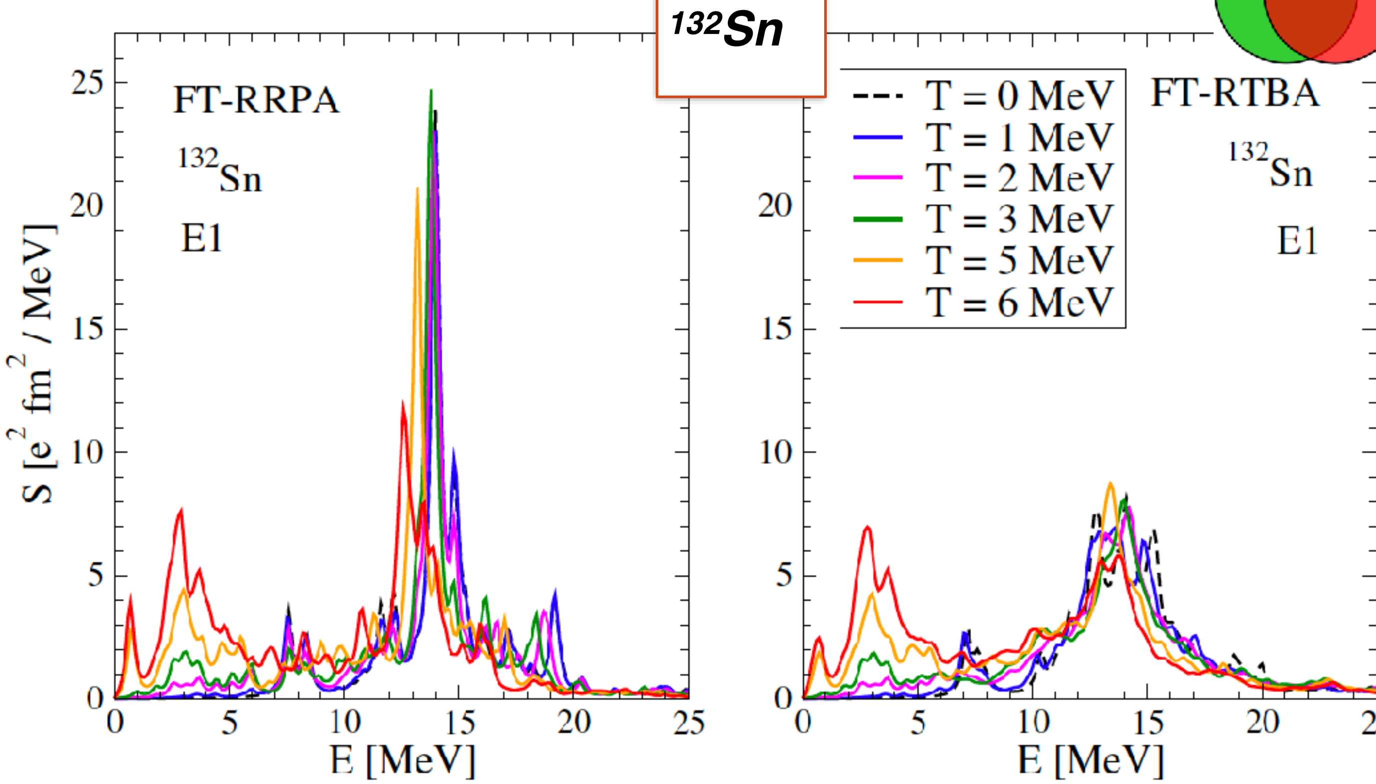
$$\Phi^{(ph,ph)}_{14,23}(\omega) = \sum_\mu \times \\ \times\Big[\delta_{13}\sum_6\frac{\gamma^\mu_{62}\gamma^{\mu*}_{64}}{\omega-\varepsilon_1+\varepsilon_6-\Omega_\mu} + \\ +\delta_{24}\sum_5\frac{\gamma^\mu_{15}\gamma^{\mu*}_{35}}{\omega-\varepsilon_5+\varepsilon_2-\Omega_\mu} - \\ -\frac{\gamma^\mu_{13}\gamma^{\mu*}_{24}}{\omega-\varepsilon_3+\varepsilon_2-\Omega_\mu} - \\ -\frac{\gamma^{\mu*}_{31}\gamma^\mu_{42}}{\omega-\varepsilon_1+\varepsilon_4-\Omega_\mu}\Big]$$

The GDR collectivity puzzle:
Dipole strength at finite temperature (T>0): ^{48}Ca and ^{132}Sn

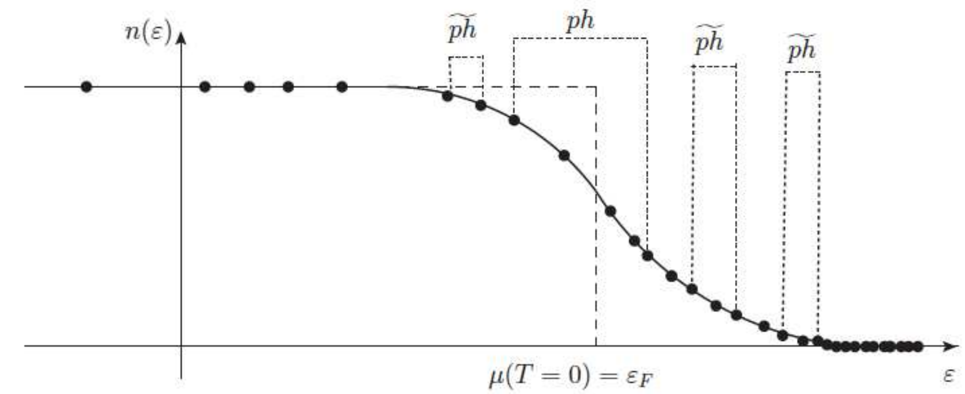
Static only (FT-REOM1) ***Static + dynamic (FT-REOM2)***







Thermal unblocking mechanism (simplified):



0th approximation:
Uncorrelated propagator

$$\tilde{R}^0_{14,23}(\omega) = \delta_{13}\delta_{24}\frac{n_2 - n_1}{\omega - \varepsilon_1 + \varepsilon_2}$$

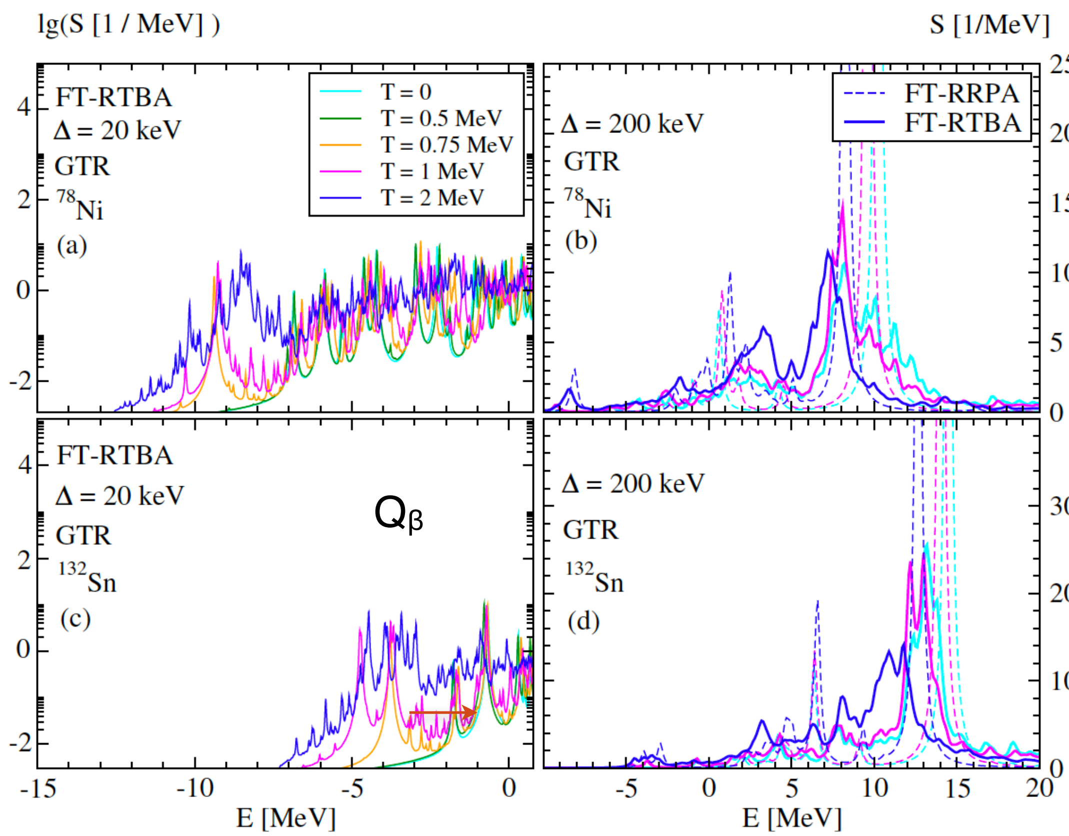
- *New transitions due to the thermal unblocking effects*
- *More collective and non-collective modes contribute to the PVC self-energy (~400 modes at T=5-6 MeV)*
- *Broadening of the resulting GDR spectrum*
- *Development of the low-energy part => a feedback to GDR*

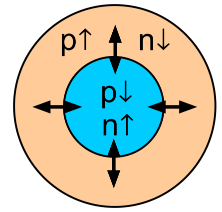
- *The spurious translation mode is properly decoupled as the mean field is modified consistently*
- *The role of the new terms in the Φ amplitude increases with temperature*
- *The role of dynamical correlations and fragmentation remain significant in the high-energy part*

E.L., H. Wibowo, Phys. Rev. Lett. 121, 082501 (2018)
H. Wibowo, E.L., Phys. Rev. C 100, 024307 (2019)

Spin-Isospin response and beta decay in stellar environments (T>0)

Gamow-Teller GT⁻ response of ^{78}Ni and ^{132}Sn



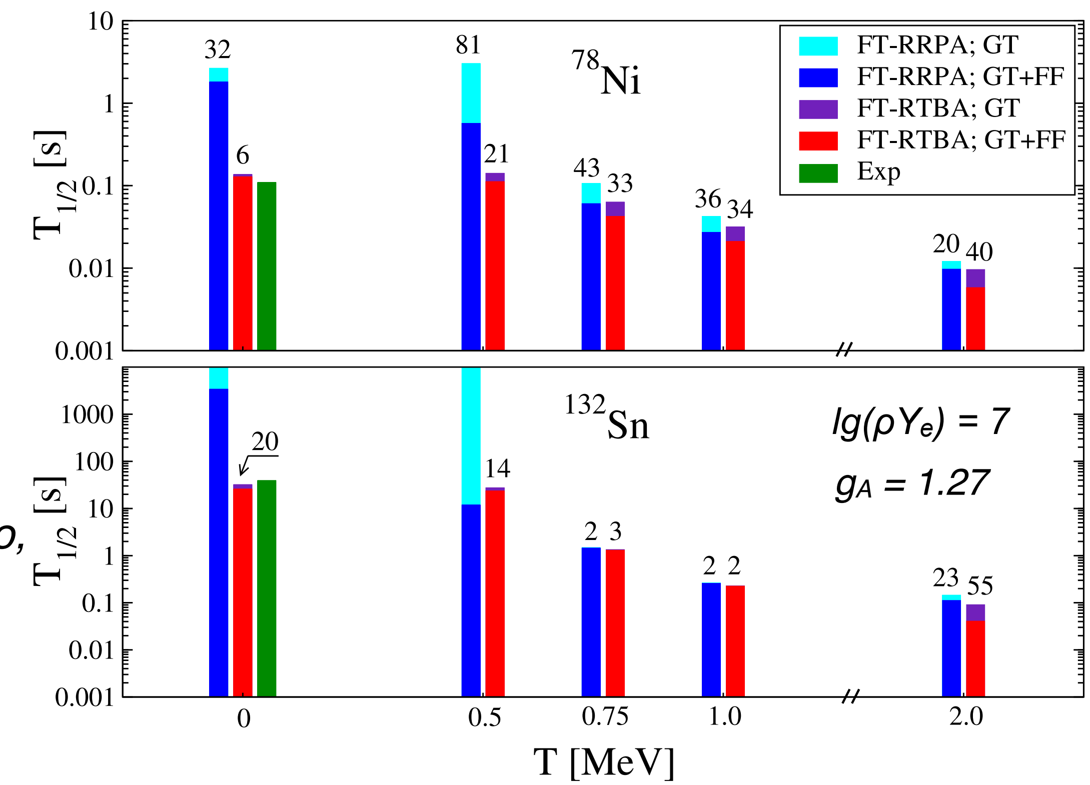


Thermal unblocking mechanism: similar but with proton-neutron pairs

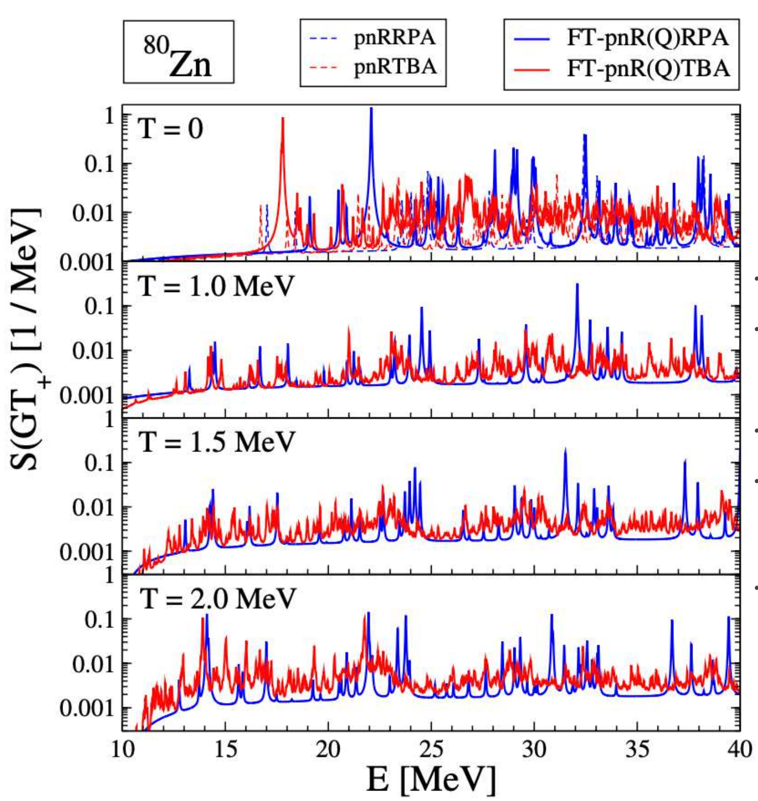
Beta decay in r-process at T > 0

E. L., C. Robin, H. Wibowo, PLB 800, 135134 (2020)

Beta decay half-lives in a stellar environment



GT+ response around ^{78}Ni

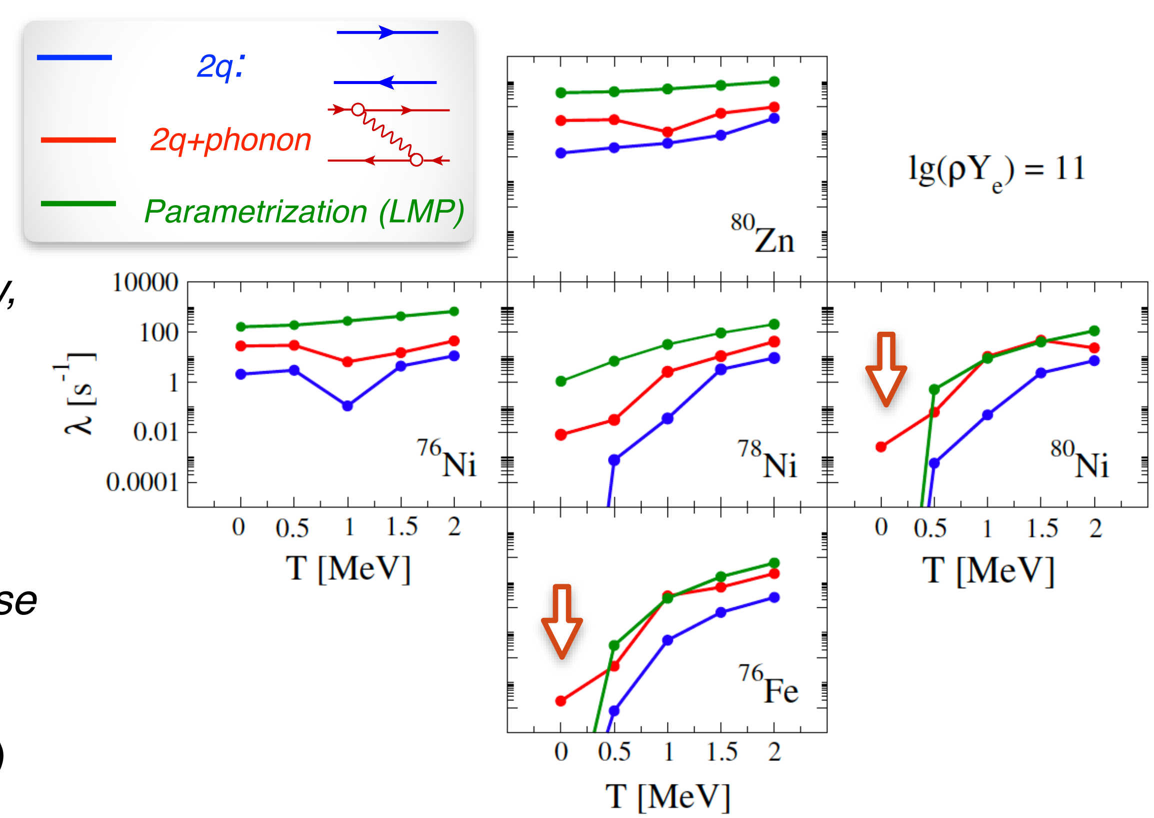


Interplay *of superfluidity and collective effects in core-collapse supernovae:*

- *Amplifies the EC rates and, consequently,*
- *Reduces the electron-to-baryon ratio, leading to lower pressure*
- *Promotes the gravitational collapse*
- *Increases the neutrino flux and effective cooling*
- *Allows heavy nuclei to survive the collapse*

E.L., C. Robin, PRC 103, 024326 (2021)

Electron capture rates around ^{78}Ni



Complete T=0 and T>0 superfluid formalism for the $G^{(2)}$ $G^{(2)}$ factorization

Theory is formulated in the HFB basis keeping 4x4 block matrix structure

S. Bhattacharjee, E.L., arXiv:2412.20751

Correlated 2q-propagator with Matsubara frequencies

$$\hat{\mathcal{R}}_{\mu\mu'\nu\nu'}(\omega_n) = \hat{\mathcal{R}}^0_{\mu\mu'\nu\nu'}(\omega_n) + \frac{1}{4}\sum_{\gamma\gamma'\delta\delta'}\hat{\mathcal{R}}^0_{\mu\mu'\gamma\gamma'}(\omega_n)\hat{\mathcal{K}}_{\gamma\gamma'\delta\delta'}(\omega_n)\hat{\mathcal{R}}_{\delta\delta'\nu\nu'}(\omega_n)$$

$$\mathcal{R}^{[lk]}_{\mu\mu'\nu\nu'}(\omega) = \sum_{fi}\sum_{\sigma=\pm}\sigma p_i \frac{\mathcal{Z}^{if[l\sigma]}_{\mu\mu'}\mathcal{Z}^{if[k\sigma]*}_{\nu\nu'}}{\omega - \sigma\omega_{fi}}$$

$$\mathcal{Z}^{if[1+]}_{\mu\mu'} = \mathcal{X}^{if}_{\mu\mu'} \qquad \mathcal{Z}^{if[2+]}_{\mu\mu'} = \mathcal{Y}^{if}_{\mu\mu'}$$
$$\mathcal{Z}^{if[3+]}_{\mu\mu'} = \mathcal{U}^{if}_{\mu\mu'} \qquad \mathcal{Z}^{if[4+]}_{\mu\mu'} = \mathcal{V}^{if}_{\mu\mu'}$$
$$\mathcal{Z}^{if[1-]}_{\mu\mu'} = \mathcal{Y}^{if*}_{\mu\mu'} \qquad \mathcal{Z}^{if[2-]}_{\mu\mu'} = \mathcal{X}^{if*}_{\mu\mu'}$$
$$\mathcal{Z}^{if[3-]}_{\mu\mu'} = \mathcal{V}^{if*}_{\mu\mu'} \qquad \mathcal{Z}^{if[4-]}_{\mu\mu'} = \mathcal{U}^{if*}_{\mu\mu'}$$

Uncorrelated 2q-propagator

$$\hat{\mathcal{R}}^0_{\mu\mu'\nu\nu'}(\omega_n) = [\omega_n - \hat{\Sigma}_3\hat{E}_{\mu\mu'}]^{-1}\hat{\mathcal{N}}_{\mu\mu'\nu\nu'}$$

The norm matrix

$$\hat{\mathcal{N}}_{\mu\mu'\nu\nu'} = \left\langle \begin{pmatrix} [A_{\mu\mu'}, A^\dagger_{\nu\nu'}] & [A_{\mu\mu'}, A_{\nu\nu'}] & [A_{\mu\mu'}, C^\dagger_{\nu\nu'}] & [A_{\mu\mu'}, C_{\nu\nu'}] \\ [A^\dagger_{\mu\mu'}, A^\dagger_{\nu\nu'}] & [A^\dagger_{\mu\mu'}, A_{\nu\nu'}] & [A^\dagger_{\mu\mu'}, C^\dagger_{\nu\nu'}] & [A^\dagger_{\mu\mu'}, C_{\nu\nu'}] \\ [C_{\mu\mu'}, A^\dagger_{\nu\nu'}] & [C_{\mu\mu'}, A_{\nu\nu'}] & [C_{\mu\mu'}, C^\dagger_{\nu\nu'}] & [C_{\mu\mu'}, C_{\nu\nu'}] \\ [C^\dagger_{\mu\mu'}, A^\dagger_{\nu\nu'}] & [C^\dagger_{\mu\mu'}, A_{\nu\nu'}] & [C^\dagger_{\mu\mu'}, C^\dagger_{\nu\nu'}] & [C^\dagger_{\mu\mu'}, C_{\nu\nu'}] \end{pmatrix} \right\rangle$$

Complete qPVC ground-state correlations

$$\mathcal{X}^{if}_{\mu\mu'} = \langle i|\alpha_{\mu'}\alpha_\mu|f\rangle \qquad \mathcal{Y}^{if}_{\mu\mu'} = \langle i|\alpha^\dagger_\mu\alpha^\dagger_{\mu'}|f\rangle$$
$$\mathcal{U}^{if}_{\mu\mu'} = \langle i|\alpha^\dagger_\mu\alpha_{\mu'}|f\rangle \qquad \mathcal{V}^{if}_{\mu\mu'} = \langle i|\alpha^\dagger_{\mu'}\alpha_\mu|f\rangle$$

$$\langle O\rangle = \sum_i p_i\langle i|O|i\rangle \qquad p_i = \frac{e^{-E_i/T}}{\sum_j e^{-E_j/T}}$$

The dynamical kernel in the factorized form:

$$\mathcal{K}^{r[11]cc(a)}_{\mu\mu'\nu\nu'}(\omega_k) = \sum_{\gamma\delta nm} p_{i_n}p_{i_m}\left(1 - e^{-(\omega_n+\omega_m)/T}\right)$$
$$\times\Big[\frac{(\Gamma^{(11)n}_{\mu\gamma}\mathcal{X}^m_{\mu'\gamma} + \Gamma^{(20)n}_{\mu\gamma}\mathcal{V}^m_{\mu'\gamma})(\mathcal{X}^{m*}_{\nu'\delta}\Gamma^{(11)n*}_{\nu\delta} + \mathcal{V}^{m*}_{\nu'\delta}\Gamma^{(20)n*}_{\nu\delta})}{\omega_k - \omega_n - \omega_m}$$
$$- \frac{(\Gamma^{(11)\bar{n}}_{\mu\gamma}\mathcal{X}^{\bar{m}}_{\mu'\gamma} + \Gamma^{(20)\bar{n}}_{\mu\gamma}\mathcal{V}^{\bar{m}}_{\mu'\gamma})(\mathcal{X}^{\bar{m}*}_{\nu'\delta}\Gamma^{(11)\bar{n}*}_{\nu\delta} + \mathcal{V}^{\bar{m}*}_{\nu'\delta}\Gamma^{(20)\bar{n}*}_{\nu\delta})}{\omega_k + \omega_n + \omega_m}\Big]$$

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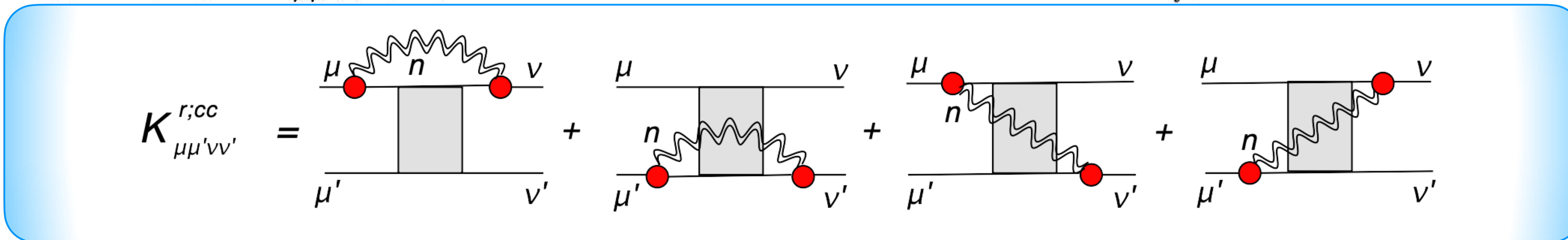
$$\hat{\mathcal{R}}_{\mu\mu'\nu\nu'}(\omega_n) = \hat{\mathcal{R}}^0_{\mu\mu'\nu\nu'}(\omega_n) +$$



$$\mathcal{Z}^{if[3-]}_{\mu\mu'} = \mathcal{V}^{if*}_{\mu\mu'} \qquad \mathcal{Z}^{if[4-]}_{\mu\mu'} = \mathcal{U}^{if*}_{\mu\mu'}$$

Complete qPVC ground-state correlations

$$\mathcal{X}^{if}_{\mu\mu'} = \langle i|\alpha_{\mu'}\alpha_{\mu}|f\rangle \qquad \mathcal{Y}^{if}_{\mu\mu'} = \langle i|\alpha^{\dagger}_{\mu}\alpha^{\dagger}_{\mu'}|f\rangle$$

$$\mathcal{U}^{if}_{\mu\mu'} = \langle i|\alpha^{\dagger}_{\mu}\alpha_{\mu'}|f\rangle \qquad \mathcal{V}^{if}_{\mu\mu'} = \langle i|\alpha^{\dagger}_{\mu'}\alpha_{\mu}|f\rangle$$

The norm matrix

$$\hat{\mathcal{N}}_{\mu\mu'\nu\nu'} = \left\langle \begin{pmatrix} [A_{\mu\mu'}, A^{\dagger}_{\nu\nu'}] & [A_{\mu\mu'}, A_{\nu\nu'}] & [A_{\mu\mu'}, C^{\dagger}_{\nu\nu'}] & [A_{\mu\mu'}, C_{\nu\nu'}] \\ [A^{\dagger}_{\mu\mu'}, A^{\dagger}_{\nu\nu'}] & [A^{\dagger}_{\mu\mu'}, A_{\nu\nu'}] & [A^{\dagger}_{\mu\mu'}, C^{\dagger}_{\nu\nu'}] & [A^{\dagger}_{\mu\mu'}, C_{\nu\nu'}] \\ [C_{\mu\mu'}, A^{\dagger}_{\nu\nu'}] & [C_{\mu\mu'}, A_{\nu\nu'}] & [C_{\mu\mu'}, C^{\dagger}_{\nu\nu'}] & [C_{\mu\mu'}, C_{\nu\nu'}] \\ [C^{\dagger}_{\mu\mu'}, A^{\dagger}_{\nu\nu'}] & [C^{\dagger}_{\mu\mu'}, A_{\nu\nu'}] & [C^{\dagger}_{\mu\mu'}, C^{\dagger}_{\nu\nu'}] & [C^{\dagger}_{\mu\mu'}, C_{\nu\nu'}] \end{pmatrix} \right\rangle$$

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$$\times \Big[\frac{(\Gamma^{(11)n}_{\mu\gamma}\mathcal{X}^{m}_{\mu'\gamma} + \Gamma^{(20)n}_{\mu\gamma}\mathcal{V}^{m}_{\mu'\gamma})(\mathcal{X}^{m*}_{\nu'\delta}\Gamma^{(11)n*}_{\nu\delta} + \mathcal{V}^{m*}_{\nu'\delta}\Gamma^{(20)n*}_{\nu\delta})}{\omega_k - \omega_n - \omega_m}$$
$$- \frac{(\Gamma^{(11)\bar{n}}_{\mu\gamma}\mathcal{X}^{\bar{m}}_{\mu'\gamma} + \Gamma^{(20)\bar{n}}_{\mu\gamma}\mathcal{V}^{\bar{m}}_{\mu'\gamma})(\mathcal{X}^{\bar{m}*}_{\nu'\delta}\Gamma^{(11)\bar{n}*}_{\nu\delta} + \mathcal{V}^{\bar{m}*}_{\nu'\delta}\Gamma^{(20)\bar{n}*}_{\nu\delta})}{\omega_k + \omega_n + \omega_m}\Big]$$

Outlook

Summary:

- RNFT is heading toward a fully self-consistent approach for describing nuclear excited states in large model spaces with a shell-model quality
- The emergent collective effects associated with the dynamical kernels of the fermionic EOMs renormalize interactions in correlated media and underlie the spectral fragmentation mechanisms
- A hierarchy of converging growing-complexity approximations generates solutions of growing accuracy and can quantify the uncertainties of the many-body theory
- The recently enabled capabilities: a complete superfluid response theory is formulated for $G^{(2)}$ $G^{(2)}$ factorized dynamical kernels at $T=0$ and $T>0$: complex ground state correlations are addressed systematically

Open theoretical problems:

- Correct separation of and delicate relationship between the static and dynamical kernels in the practical approximate solutions (linking bare and effective interactions)
- Consistency between the direct and pairing channels in the dynamical kernels in practical implementations
- Ambiguities of numerical implementations on the 2p2h and 3p3h levels of complexity (complex configurations in pairing and beyond)
- An adequate assessment of the quantitative role of fully connected multi-fermion correlators (~ irreducible three- and many-body “forces” in the medium).
- …

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Manqoba Hlatshwayo (NQCC UK)
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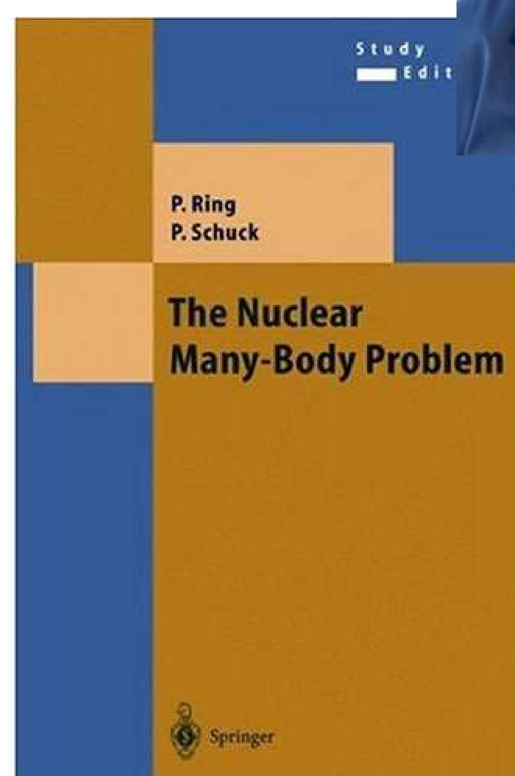
Peter Schuck (IPN Orsay)
Peter Ring (TU München)

And beyond

US-NSF PHY-2209376

US-NSF PHY-2515056



John Novak

Kyle Morrisey

Sumit Bhattacharjee

Thank you!