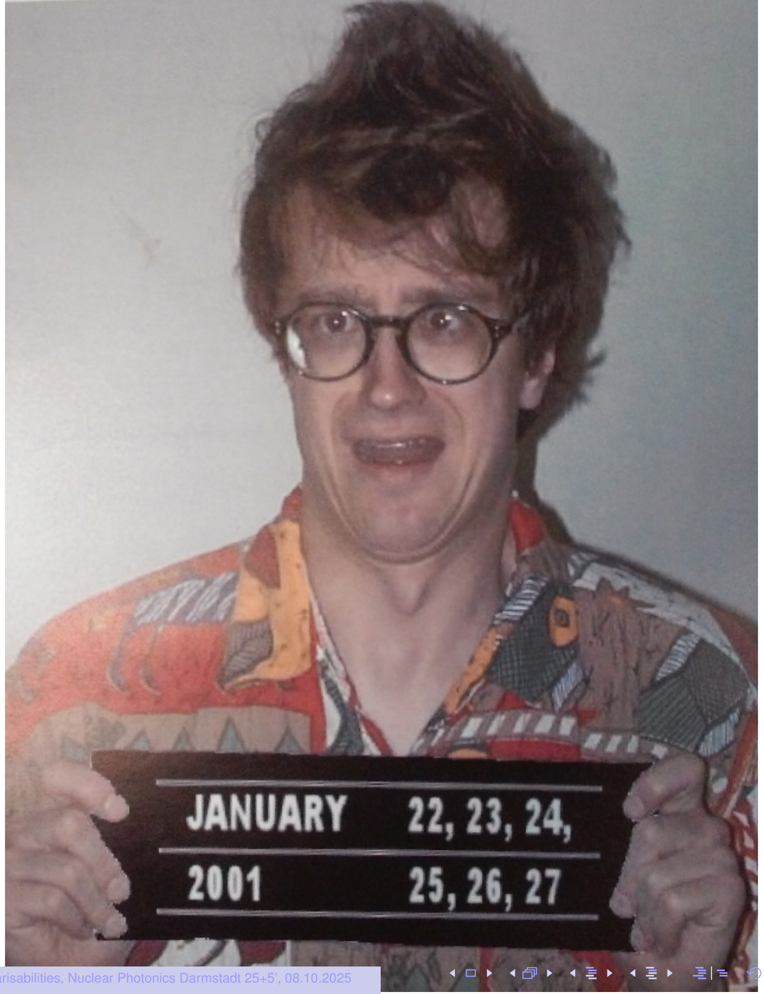

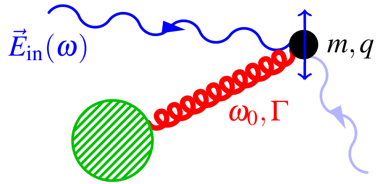

Compton Scattering Reveals Garment Composition & Fashion Mind

A Theoretically Grand Tour of
Compton Scattering and Nucleon Polarisabilities

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Institute for Nuclear Studies
The George Washington University, DC, USA

with **Alex Long & Junjie Liao (GW)**,
Judith A. McGovern (U. Manchester),
Andreas Nogga, Xiang-Xiang Sun (FZ Jülich),
Daniel R. Phillips (Ohio U.)

GW INS
Institute for Nuclear Studies
THE GEORGE WASHINGTON UNIVERSITY
WASHINGTON, DC

Office of Science
U.S. DEPARTMENT OF ENERGY

How do constituents of the nucleon react to external fields?

How to reliably extract proton, neutron, spin polarisabilities?

Community Effort: Plan effective experiments & test theory!

Office of Nuclear Physics
Exploring Nuclear Matter - Quarks to Stars

Exp-Th Compton Roadmap in "Next-Gen γ Source": IJMPG49 (2022) 010502

Review: +G. Feldman: Prog. Part. Nucl. Phys. 67 (2012) 841

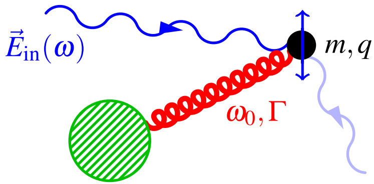
Transition Density Formalism and ^{3}He: hg/JMcG/AN/DRP: Few-Body Syst. 61 (2020) 61 [2005.12207] [nucl-th]

^{4}He $\mathcal{O}(e^2\delta^3)$ Compton & TDA Digest: Liao/hg/JMcG/AN/DRP: EPJA 60 (2024) 132 [2401.16995] [nucl-th]

^{6}Li & SRG-And-Back: Long/hg/AN/Sun in preparation

(a) Polarisabilities: Stiffness of Charged Constituents in El.- Mag. Fields

Example: induced electric dipole radiation from harmonically bound charge, damping Γ Lorentz/Drude 1900/1905



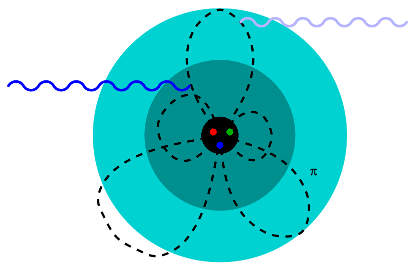
$$\vec{d}_{\text{ind}}(\omega) = \underbrace{\frac{q^2}{m}\,\frac{1}{\omega_0^2-\omega^2-\mathrm{i}\Gamma\omega}}_{=:\,4\pi\,\alpha_{E1}(\omega)}\;\vec{E}_{\text{in}}(\omega)$$

$=:\,4\pi\,\alpha_{E1}(\omega)$ "displaced volume" $[10^{-4}\ \text{fm}^3]$

Energy- (ω)-dep. multipoles dis-entangle *interaction scales, symmetries & mechanisms* of constituents.

Clean, perturbative probe: χiral symmetry of pion-cloud & its breaking, $\Delta(1232)$, spin-constituents.

Fundamental hadron properties, like charge, mass, mag. moment, $\langle r_N^2\rangle\ldots$ PDG



(b) Dissecting Theories: Compton Multipoles

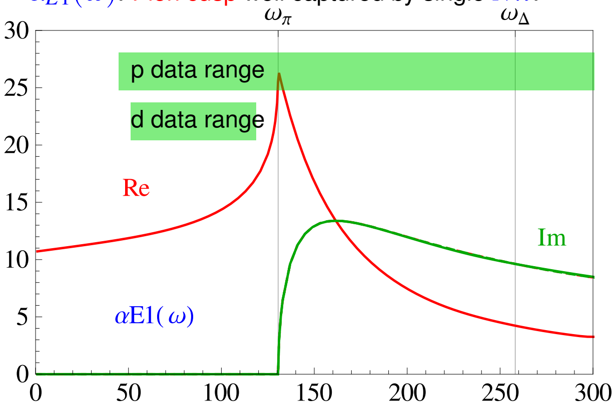
Full amplitudes for observables/fits, but *multipoles* to reveal two-photon response of constituents.

$$T = \underbrace{\begin{bmatrix}\text{point spin-}\frac{1}{2} + \pi^0 \text{ pole} \\ + \text{anom. mag. moment}\end{bmatrix}}_{\text{Powell amplitudes}} + 2\pi \Big[\underbrace{\alpha_{E1}(\omega)\vec{E}^2 + \beta_{M1}(\omega)\vec{B}^2}_{\text{2 scalar dipoles: electric, magnetic}} \underbrace{+\gamma_{E1E1}(\omega)\vec{\sigma}\cdot(\vec{E}\times\dot{\vec{E}}) + \gamma_{M1M1}(\omega)\vec{\sigma}\cdot(\vec{B}\times\dot{\vec{B}}) + \dots}_{\text{4 spin dipoles}} + \dots \Big]$$

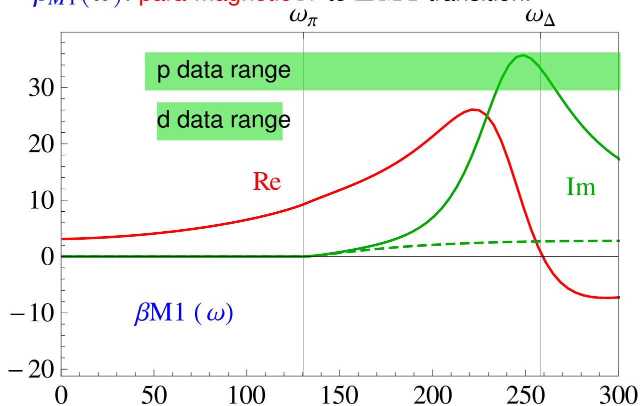
***Polarisabilities*: *Energy-dependent Multipoles* of real Compton scattering.**

$\Longrightarrow$ "Static" α_{E1} etc. compress rich dynamics into few numbers. $\Longrightarrow$ Need Theory to extrapolate!

$\alpha_{E1}(\omega)$: Pion cusp well captured by single-$N\pi$.



$\beta_{M1}(\omega)$: para-magnetic N-to-Δ $M1$-transition.



(c) The Polarisabilities Project: Relevant Data and Reliable Theory

2023 US & 2024 EU LRPs: Polarisabilities benchmark low-energy hadron structure.

[Since last US LRP,] **substantial progress has been made [. . .], with strong international efforts and synergistic advancements in experiment and theory.**

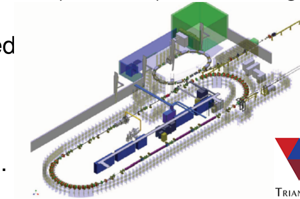
***The Present and Future of QCD*, US Town Meeting White Paper 2023**

Compton Community Goal: Unified framework with reliable error bars for proton, deuteron, ^{3}He, ^{4}He, . . . (elastic & inelastic) into $\Delta(1232)$ region.

Exp-Th Compton Roadmap in "Next-Gen γ Source" Int. J. Mod. Phys. G49 (2022) 010502 [2012.10843]

Lattice QCD: relate to fundamental interactions $\longrightarrow$ *polarQCD* (Alexandru/Lee) 2005-; *NPLQCD* 2006-; *LHPC* (Engelhardt) 2007-; Leinweber/. . . (Adelaide) 2013; Wang/. . . (Beijing) 2024-

Experiment: Significant investments; data taken/scheduled/approved



COMPTON@HIγS (TUNL/Duke U. NC, USA; DOE):

> 3000 hrs already committed at $60 - 110$ MeV

proton doubly & beam pol. deuteron unpol & beam pol.

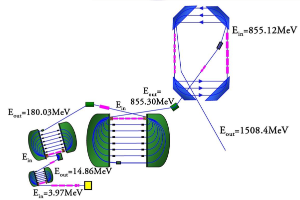
^{3}He unpol & doubly pol. ^{4}He, ^{6}Li unpol.

A2 @ MAMI (Mainz U. Germany; DFG: 5-year SFB):

running, data cooking and planned

proton $100 - 400$ MeV: beam & target pol.

deuteron, ^{3}He, ^{4}He unpol., beam & target pol.

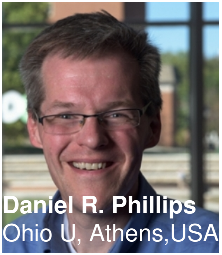


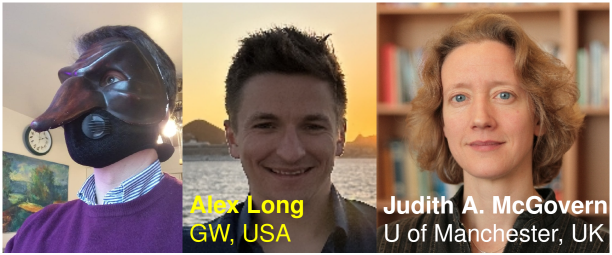
MAXlab (Lund U., Sweden): data cooking continues deuteron $100 - 160$ MeV: unpol.

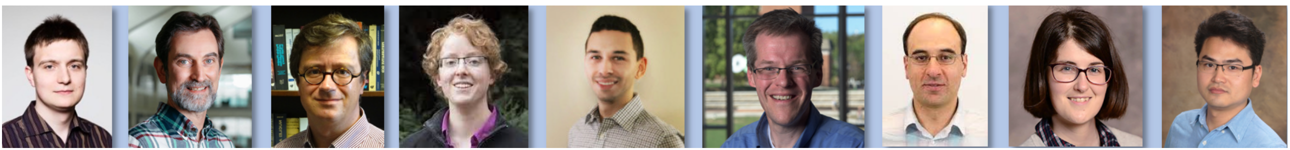
Chiral EFT: data consistency, binding effects, analysis, extraction

(d) Our Theory Collaboration: χEFT With Error Bars for Nuclear Physics!

Goals: Comprehensive picture of Compton scattering and nucleon polarisabilities,
with probabilistic interpretation of theory truncation uncertainties.
Guide, support, analyse, predict new generation of experiments, and relate data and lattice QCD.





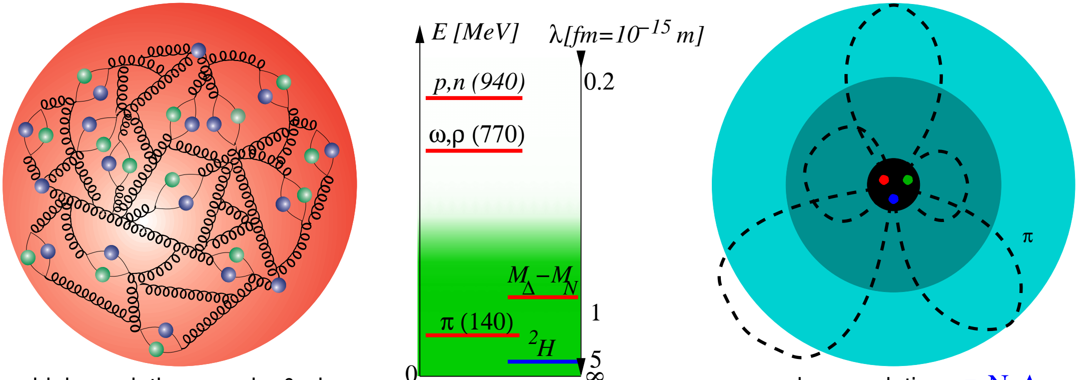


The BUQEYE Collaboration: Bayesian Uncertainty Quantification: Errors in Your EFT

. . . and, over the years, . . .

Berhan T. Demissie, Hao Wang, Junjie Liao (GW), **Bruno Strandberg** (U Glasgow), **Arman Margaryan** (Duke, **Deepshikha Choudhury Shukla** (Ohio U & GW), **Robert Hildebrandt**, Thomas R. Hemmert (TU München)

(a) The Low-Energy Method: Chiral Effective Field Theory



high resolution: quarks & gluons

low resolution: π, N, Δ

Degrees of freedom $\pi, N, \Delta(1232)$ **+ all interactions allowed by symmetries:** Chiral SSB, gauge, iso-spin,...

$\Longrightarrow$ **Chiral Effective Field Theory χEFT $\equiv$ low-energy QCD**

$$\mathcal{L}_{\chi\text{EFT}} = (D_\mu \pi^a)(D^\mu \pi^a) - m_\pi^2\, \pi^a \pi^a + \dots + N^\dagger [\mathrm{i}\, D_0 + \frac{\vec{D}^2}{2M} + \frac{g_A}{2f_\pi}\, \vec{\sigma} \cdot \vec{D}\pi + \dots] N + C_0 \left(N^\dagger N\right)^2 + \dots$$

Controlled expansion in $\delta = \frac{M_\Delta - M_N}{\Lambda_\chi} \approx \sqrt{\frac{m_\pi}{\Lambda_\chi}} \approx 0.4 \ll 1$ (numerical fact) Pascalutsa/Phillips 2002 $\Longrightarrow$ **Model-independent, uncertainty estimate.**

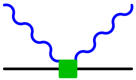
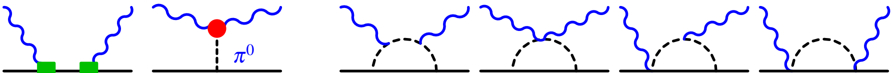
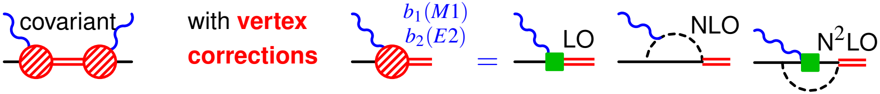
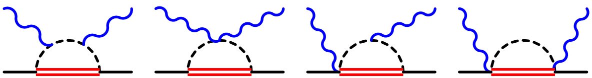
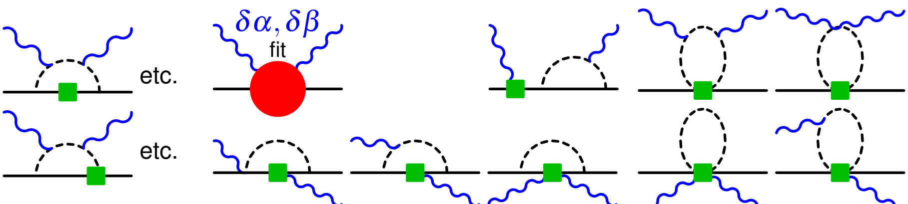
(b) All 1N Contributions to N^{4}LO

Bernard/Kaiser/Meißner 1992-4, Butler/Savage/Springer 1992-3, Hemmert/. . . 1998
McGovern 2001, hg/Hemmert/Hildebrandt/Pasquini 2003
McGovern/Phillips/hg 2013

Unified Amplitude: gauge & RG invariant set of all contributions which are

in low régime $\omega \lesssim m_\pi$ at least N^{4}LO ($e^2\delta^4$): accuracy $\delta^5 \lesssim 2\%$;

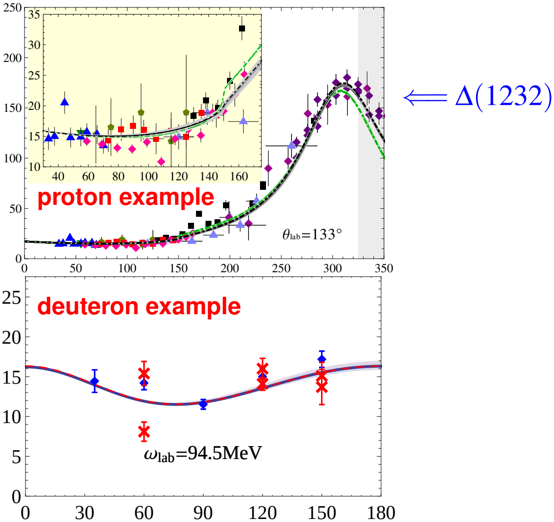
or in high régime $\omega \sim M_\Delta - M_N$ at least NLO ($e^2\delta^0$): accuracy $\delta^2 \lesssim 20\%$.

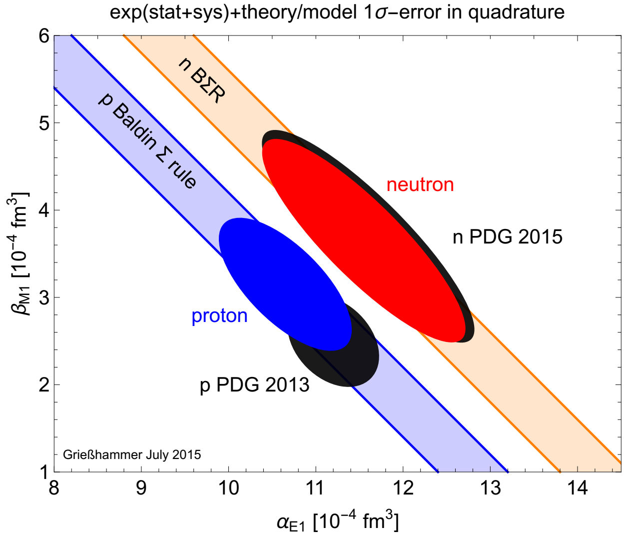
	$\omega \lesssim m_\pi$	$\omega \sim M_\Delta - M_N \approx 300$ MeV
 Thomson term: $-\frac{Z^2\alpha_{EM}}{M}$	$e^2\delta^0$ ***LO***	$e^2\delta^0$ ↘NLO
	$e^2\delta^2$ N^{2}LO	$e^2\delta^1$ N^{2}LO
	$e^2\delta^3$ N^{3}LO	$e^2\delta^{-1}$ ↗***LO***
	$e^2\delta^3$ N^{3}LO	$e^2\delta^1$ N^{2}LO
	$e^2\delta^4$ N^{4}LO	$e^2\delta^2$ N^{3}LO

Unknowns: short-distance $\delta\alpha, \delta\beta \Longleftrightarrow$ static α_{E1}, β_{M1} (offset) $\Longrightarrow$ ω-dependence *predicted.*

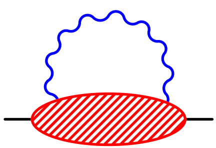
(c) Scalar Polarisabilities from Consistent p & d Databases

database: JMcG/DRP/hg/
Feldman PPNP 2012





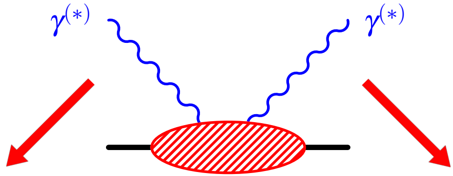
	α_{E1} $[10^{-4}\ \mathrm{fm}^3]$	β_{M1} $[10^{-4}\ \mathrm{fm}^3]$	χ^2/d.o.f.
proton (Baldin, N^{2}LO) McGovern/Phillips/hg EPJA 2013	$10.65 \pm 0.35_{\mathrm{stat}} \pm 0.2_{\Sigma} \pm 0.3_{\mathrm{theory}}$	$3.15 \mp 0.35_{\mathrm{stat}} \pm 0.2_{\Sigma} \mp 0.3_{\mathrm{theory}}$	$113.2/135$
neutron (Baldin, NLO) Compton@MAX-lab PRL 2014	$11.55 \pm 1.25_{\mathrm{stat}} \pm 0.2_{\Sigma} \pm 0.8_{\mathrm{theory}}$	$3.65 \mp 1.25_{\mathrm{stat}} \pm 0.2_{\Sigma} \mp 0.8_{\mathrm{theory}}$	$45.2/44$

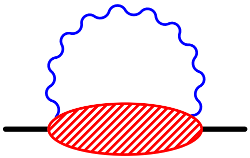


$\Longrightarrow$ **neutron ≈ proton polarisabilities:** $\alpha_{E1}^{p-n} = -0.9 \pm 1.6_{\mathrm{tot}}$ – **exp. & neutron errors dominate**

$-0.6 \pm 1.2_{\mathrm{tot}}$ PDG 2022

CottinghamΣR explains $M_\gamma^p - M_\gamma^n$ **with** $\alpha_{E1}^{p-n} = -1.7 \pm 0.4_{\mathrm{tot}}$ **Gasser/Hoferichter/Leutwyler/Rusetzky [1506.06747]**



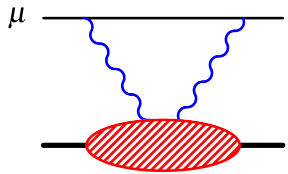


elmag. self-energy part of nucleon mass splitting

$M_\gamma^{\mathrm{p}} - M_\gamma^{\mathrm{n}} \approx [1.1 \pm 0.5]$ MeV

with $\alpha_{E1}^{p-n} = -1.7 \pm 0.4_{\mathrm{tot}}$

Hoferichter/Gasser/Leutwyler/Rusetzky 2015



Muonic H/Nucleus

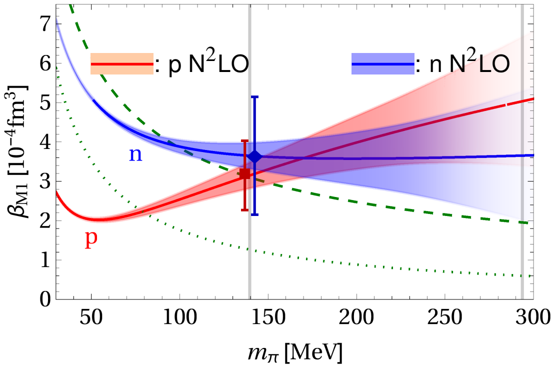
2γ contributes to Lamb shift

for proton/nucleus radius

Birse/McGovern 2012,. . .

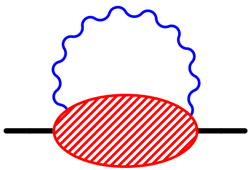
χ**EFT: explicit m_π-dependence** $\mathcal{O} = c_0(m_\pi) + c_1(m_\pi)\delta^1 + c_2(m_\pi)\delta^2 +$ **unknown** $\times\, \delta^3$**, fixed at m_π^{phys}.**

Uncertainties: Bayesian order-by-order at each m_π.



Isospin splitting *statistically* significant for $m_\pi \lesssim 120$ MeV.

$\Longrightarrow$ SPECULATION – NO ERROR BARS



Cottingham Σ Rule: $\beta_{M1}^{\text{p}-\text{n}}$ is *one of several* inputs into the proton-neutron self-energy difference:

$$M_{\text{p}-\text{n}} = M_{\text{p}-\text{n}}^{\text{strong}} + M_{\text{p}-\text{n}}^{\text{em,elastic}} - A\, \beta_{M1}^{\text{p}-\text{n}}$$

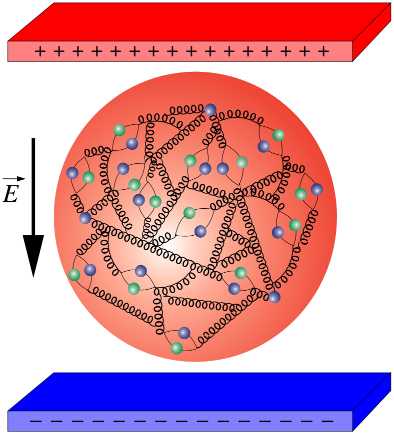
Impact on p-n mass difference?: $-A\beta_{M1}^{\text{p-n}} \approx 0.5$ MeV wants more stable n as $m_q \searrow$, competes with $M_{\text{p-n}}^{\text{strong}}$.

$\to$ Neutron lifetime $\to$ Big Bang Nucleosynthesis $\to$ Anthropic Principle?

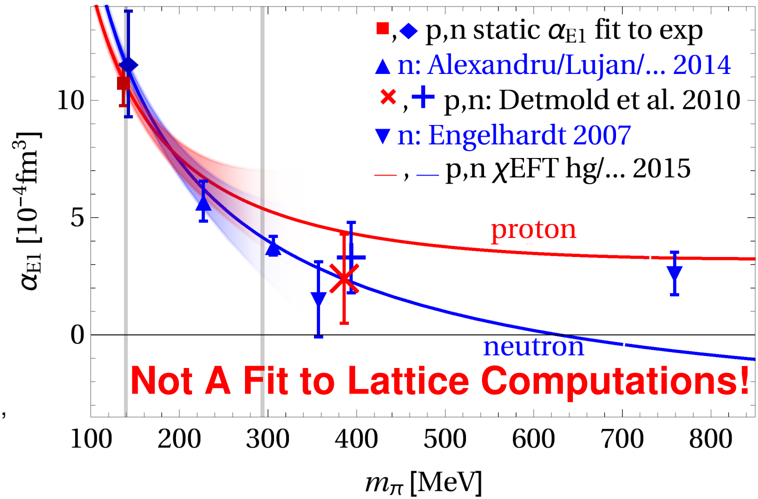
(f) Chiral Extrapolations for Polarisabilities in Lattice QCD

Towards comparable uncertainties in experiment, χEFT and lattice QCD.

χEFT: reliable error estimate for $\frac{m_\pi}{\Lambda_\chi} \ll 1$. $\Longrightarrow$ ***Fading corridors beyond*** ~ 250 MeV.



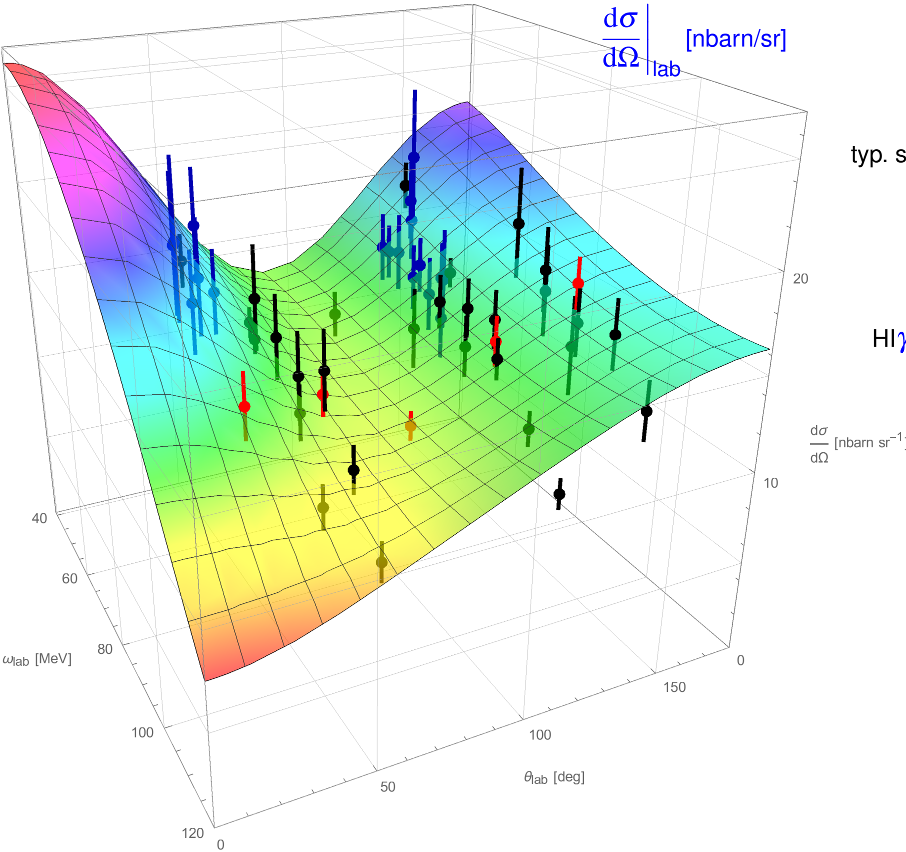
Example: static electric polarisability α_{E1}



Ongoing: charged sea, $m_\pi \searrow 200$ MeV, larger volumes, more statistics,. . .

Active lattice groups:
Alexandru/Lee/. . . 2005-;
Engelhardt/LHPC 2006-;
EMC/NPLQCD 2006-, 2015-;
Leinweber/Primer/Hall/. . . 2013-
Wang/. . . (Beijing) 2024-

(g) Getting to the Neutron: Deuteron Theory and Data



52 points Illinois, Saskatoon, Lund

typ. stat.+uncorrel. sys.: $\pm[7\dots10]\%$

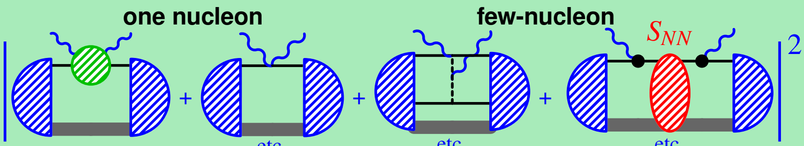
typ. correl. sys.: $\pm[3\dots5]\%$

limited energy & angle coverage

HIγS run being analysed Godagama/. . .

(h) How to Get to the Neutron?

deuteron: hg/.../+Phillips/+McGovern 2004-; **MECs:** Beane/... 1999-2005
^{3}**He:** Shukla/... 2009 +Strandberg/Margaryan/hg/... [1804.01206]
^{4}**He:** hg/.../+Nogga EPJA 60 (2024) 132 [2401.16995]



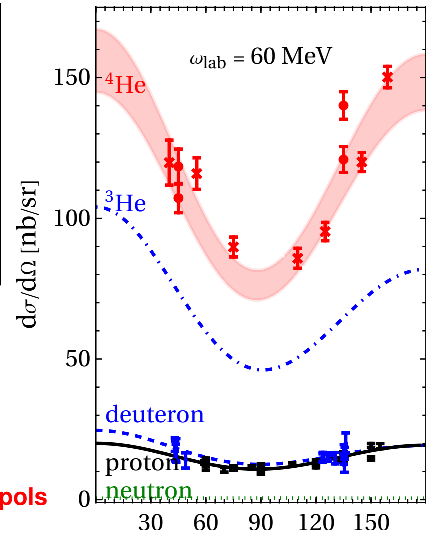
N **structure** $\alpha_{E1}, \beta_{M1}, \gamma_i$	N Born: charge & mag. moment	$\pi^\pm$ exchange **largest part**	coherent rescattering relevant for $\omega \to 0$

Parameter-free, except N polarisabilities.

Experiment: More charge & MECs $\Longrightarrow$ more counts $\Longrightarrow$ *heavier nuclei*

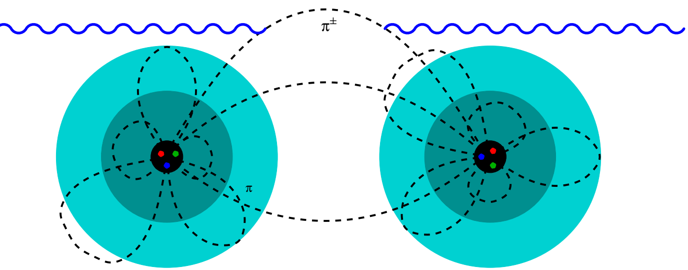
Theory: Reliable only if nuclear binding & levels accurate $\Longrightarrow$ *lighter nuclei*

***Sweet-spot*: Complementing Targets of Opportunity.**



Deuteron, ^{4}He, ^{6}Li: sensitive to $\alpha_{E1}^p + \alpha_{E1}^n$, $\beta_{M1}^p + \beta_{M1}^n$ $\Longrightarrow$ **neutron pols**

^{3}**He**: sensitive to $2\alpha_{E1}^p + \alpha_{E1}^n$, $2\beta_{M1}^p + \beta_{M1}^n$ $\Longrightarrow$ **neutron pols**



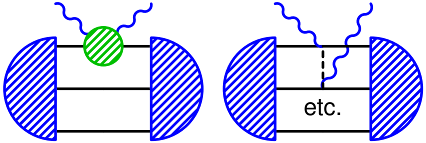
Model-independently subtract binding effects.

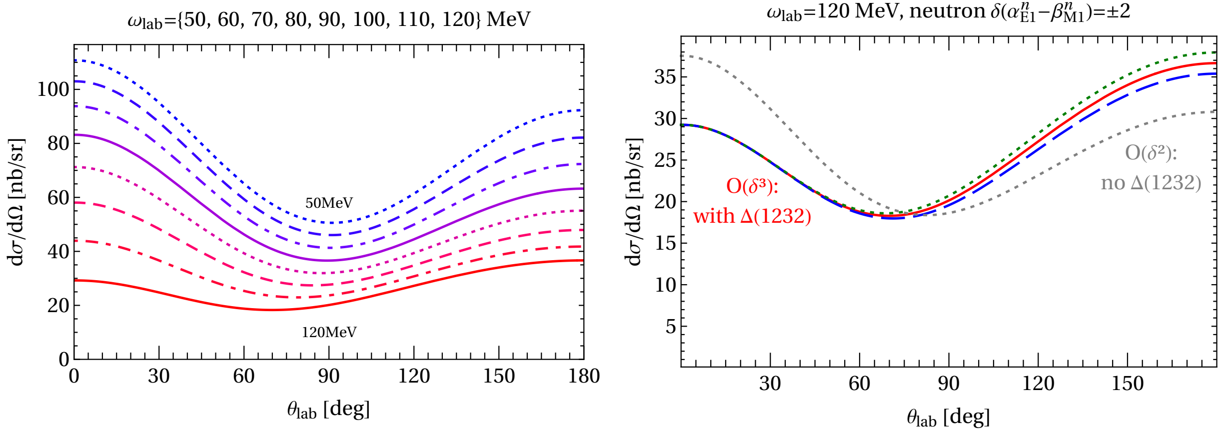
Chirally consistent 1N & few-N: potentials, wave functions, currents, π-exchange.

With increasing A, less precise polarisabilities, more charged-pion component of NN int.

(i) The Hard Way: ^{3}He Theory Took A PhD

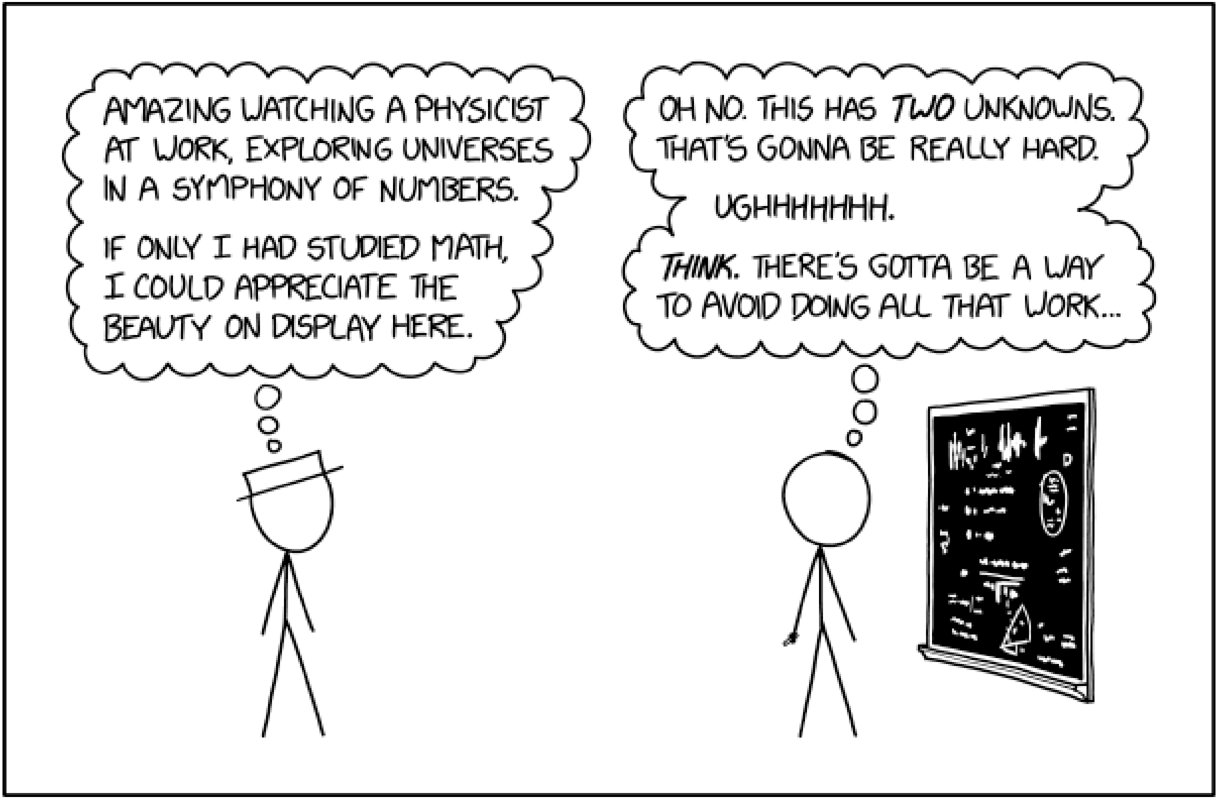
Shukla/Phillips/Nogga 2009, err. PRL **120** (2018) 249901
+Strandberg/Margaryan/hg/McG/Ph [1804.01206]
data taken at HIγS 2024, under analysis





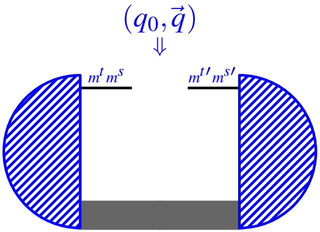
Observables available as *Mathematica* notebook.

(j) 3 Reasons To Simplify: Patterns; Reduce Computational Complexity; And. . .

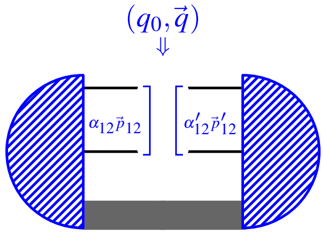


xkcd 2019

(k) Transition-Density Amplitude (TDA) Method: Idea



$A-1$ **spectator nucleons**
1 Active Nucleon:
one-body density



$A-2$ **spectator nucleons**
2 Active Nucleons:
two-body density

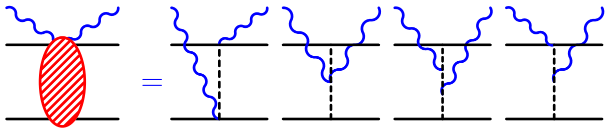
n-Body Transition Density Amplitude:

n nucleons with intrinsic momenta
and specific quantum numbers α_{12}

– absorb momentum transfer $(q_0, \vec{q})$;
– re-arrange quantum numbers to α'_{12};
– get absorbed back into nucleus.

only depends on quantum numbers of actives and mom. transfer

$\mathcal{O}(e^2\delta^2)$ 2N kernel:
Compton



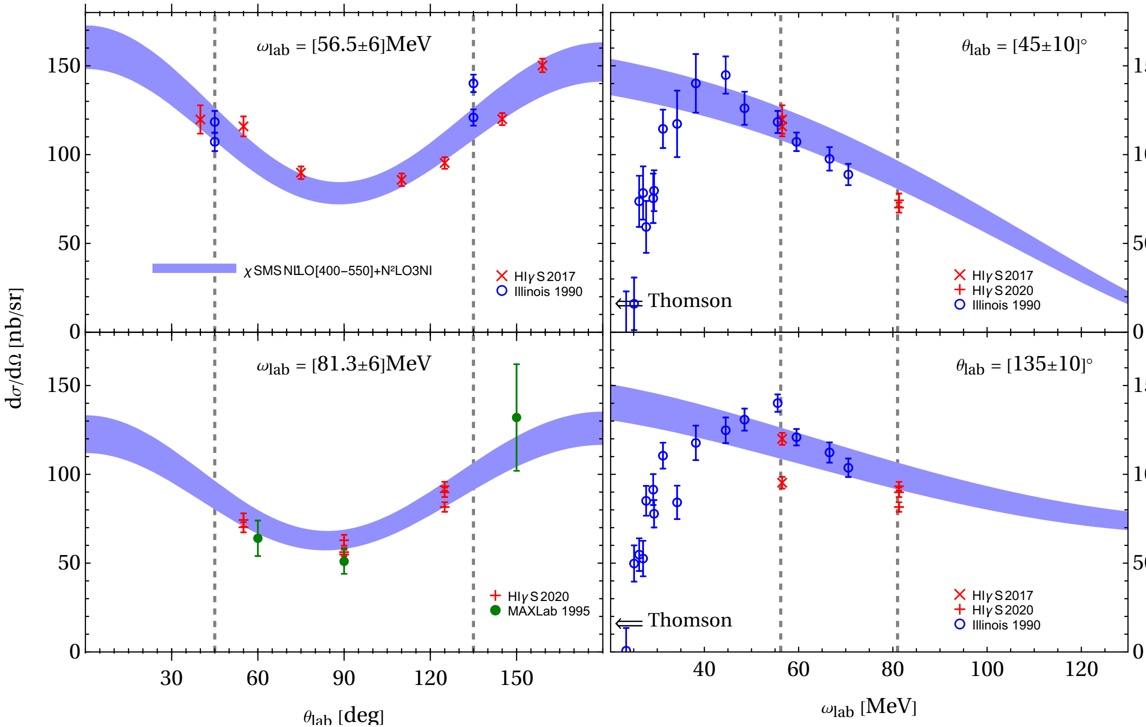
Beane/... 1999-2005

Idea: Split calculation into

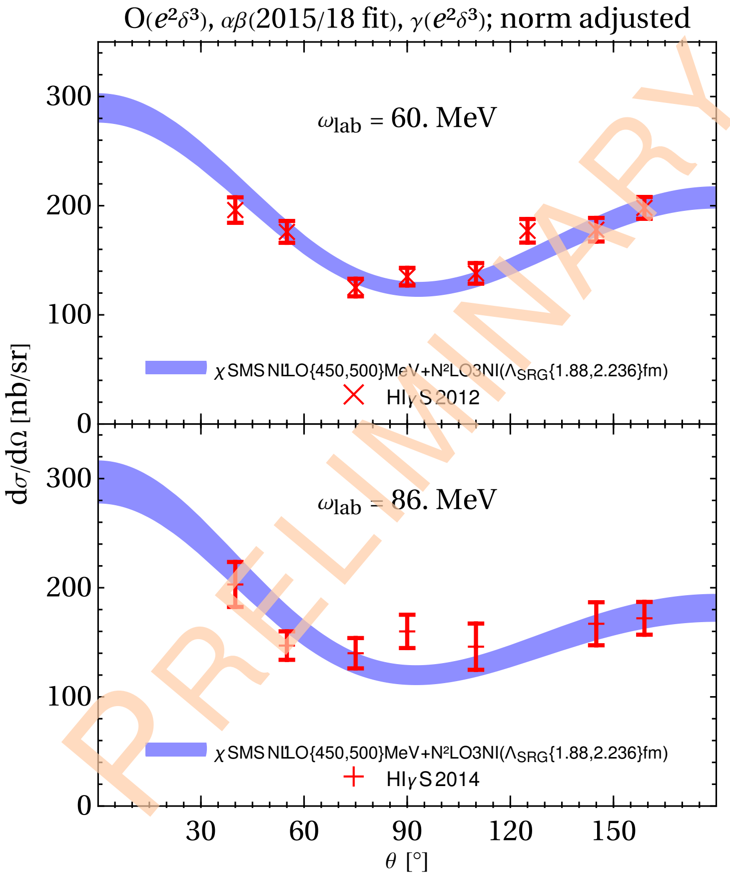
kernel: interaction with n active nucleons	|	**structure: $A-n$ spectators**
recycle same reaction for different nuclei	|	**recycle same nucleus for different reactions**
Compton on ^{3}He, ^{3}H, ^{4}He, ^{6}Li, ...	|	**^{4}He Compton, π prod., FFs, dark matter,...**

χEFT hierarchy of few-body interactions: onebody, twobody $\gg$ threebody $\gg$ fourbody...

$\mathcal{O}(e^2\delta^3)$ with established α_{E1}, β_{M1}; theory uncertainties: vary χSMS potential (dominant), numerics (tiny).



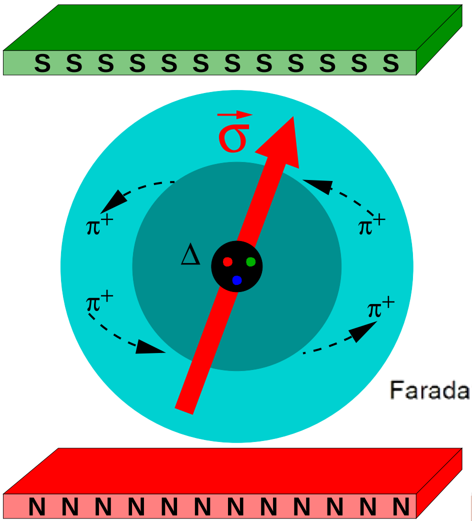
(m) ^{6}Li Compton Scattering: TDA with "SRG-And-Back" NCSM



3. Spin Polarisabilities and Nucleon Spin Structure

(a) Spin Polarisabilities: Nucleonic Bi-Refringence and Faraday Effect

Optical Activity: Response of spin-degrees of freedom, complements JLab spin programme.



$$\mathcal{L}_{\text{pol}} = 4\pi\, N^\dagger \times \Big\{ \tfrac{1}{2}\Big[\alpha_{E1}\,\vec{E}^2 + \beta_{M1}\,\vec{B}^2\Big]$$

electric / magnetic scalar dipole

$$+\tfrac{1}{2}\Big[\gamma_{E1E1}\,\vec{\sigma}\cdot(\vec{E}\times\dot{\vec{E}}) + \gamma_{M1M1}\,\vec{\sigma}\cdot(\vec{B}\times\dot{\vec{B}})$$

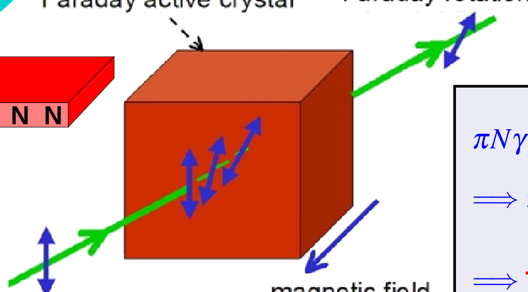
"pure" spin-dependent dipole

$$-2\,\gamma_{M1E2}\,\sigma_i\,B_j\,E_{ij} + 2\,\gamma_{E1M2}\,\sigma_i\,E_j\,B_{ij}\Big] + \dots\Big\}\,N$$

"mixed" spin-dependent dipole

+ quadrupole etc.

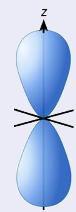
$E_{ij} := \frac{1}{2}(\partial_i E_j + \partial_j E_i)$ etc.



$\pi N\gamma:\; -\dfrac{g_A}{2f_\pi}\,\vec{\sigma}\cdot(\vec{q}+e\vec{\epsilon}) + \dots$

$\Longrightarrow$ π emission/absorption depends on N spin.

$\Longrightarrow$ **Test χiral Symmetry!**



(b) The Polarisation Plethora: p/^{3}He ($S=\frac{1}{2}$) & d/^{6}Li ($S=1$)

Babusci/... 1998
hg [1304.6594]
cf. Arenhövel/Sanzone 1991

Unpol./linear/circular beam on scalar+vector($S=\frac{1}{2}$: p/^{3}He)+tensor($S=1$: d/^{6}Li) target.

$$\left.\frac{d\sigma}{d\Omega}\right|_{\text{unpol}} \times \Bigg[\; 1 + \Sigma^{\text{lin}}(\omega,\theta)\; P_{\text{lin}}^{(\gamma)}\cos 2\phi_{\text{lin}}$$

1 beam asymmetry

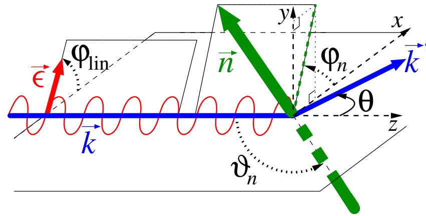
$$+ \sum_{\substack{I=1,2\\0\le M\le I}} T_{IM}(\omega,\theta)\quad P_I^{(\text{d})}\, d^I_{M0}(\theta_{\text{d}})\,\cos[M\phi_{\text{d}}-\frac{\pi}{2}\delta_{I1}]$$

target asymmetries: 1 for p/^{3}He, 4 for d/^{6}Li

$$+ \sum_{\substack{I=1,2\\0\le M\le I}} T_{IM}^{\text{circ}}(\omega,\theta)\quad P_{\text{circ}}^{(\gamma)}\, P_I^{(\text{d})}\, d^I_{M0}(\theta_{\text{d}})\,\sin[M\phi_{\text{d}}+\frac{\pi}{2}\delta_{I1}]$$

circpol. double asymmetries: 2 for p/^{3}He; 4 for d/^{6}Li
linpol. double asymmetries: 3 for p/^{3}He; 8 for d/^{6}Li

$$+ \sum_{\substack{I=1,2\\-I\le M\le I}} T_{IM}^{\text{lin}}(\omega,\theta)\quad P_{\text{lin}}^{(\gamma)}\, P_I^{(\text{d})}\, d^I_{M0}(\theta_{\text{d}})\,\cos[M\phi_{\text{d}}-2\phi_{\text{lin}}-\frac{\pi}{2}\delta_{I1}]\Bigg]$$



Differences Δ and asymmetries $\frac{\Delta}{\text{sum}}$

Sensitivity of 8 for p/^{3}He / 18 for d/^{6}Li obs. to 6 polarisabilities of 2 N in 2 kinematics (ω,θ) + additional constraints:

– scalar polarisabilities $\alpha_{E1},\ \beta_{M1}$ – $\gamma_0,\ \gamma_\pi$ (???) – **Experiment: detector settings,...**

More Beam-to-Recoil/Ejectile Polarisation transfer observables on request...

No single measurement to provide definitive answers: multi-parameter extractions, systematics, validation.

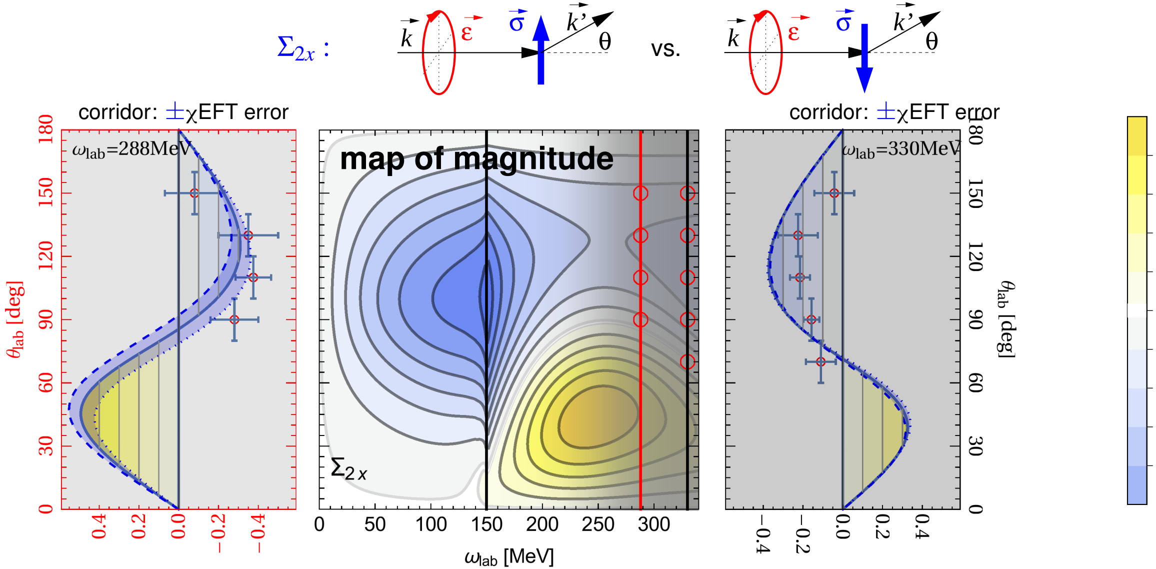
$\Longrightarrow$ **Experiment & Theory collaborate to identify *observables with biggest impact.***

(c) Proton Spin Polarisabilities with Asymmetries

$\mathcal{O}(e^2\delta^3)$: hg/Hildebrandt/...2003
$\mathcal{O}(e^2\delta^4)$: hg/McG/Ph [1511.0952]&[1711.11546]
exp MAMI A2: Martel/...PRL 2014 & Paudyak/...2019
cf. Mornacchi/...2022: more conservative errors

Incoming γ circularly polarised, sum over final states. N-spin in $(\vec{k},\vec{k}')$-plane, perpendicular to $\vec{k}$:

Σ_{2x} :



static $[10^{-4}\ \mathrm{fm}^4]$	γ_{E1E1}	γ_{M1M1}	γ_{E1M2}	γ_{M1E2}
MAMI A2 2019 proton [1909.02032]	-2.9 ± 0.5	2.7 ± 0.4	-0.85 ± 0.7	8 2.0 ± 0.4
χEFT proton **predicted**	$-1.1\pm1.9_{\mathrm{th}}$	$2.2\pm0.5_{\mathrm{stat}}\pm0.6_{\mathrm{th}}$	$-0.4\pm0.6_{\mathrm{th}}$	$1.9\pm0.5_{\mathrm{th}}$
χEFT neutron **predicted**	$-4.0\pm1.9_{\mathrm{th}}$	$1.3\pm0.5_{\mathrm{stat}}\pm0.6_{\mathrm{th}}$	$-0.1\pm0.6_{\mathrm{th}}$	$2.4\pm0.5_{\mathrm{th}}$
fixed-t DR±data Pasquini/...	-3.85 ± 0.5	2.8 ± 0.1	-0.15 ± 0.15	1.0 ± 0.1
covariant $\mathcal{O}(e^2\delta^3)$ Lensky/...	$-3.3\pm0.8_{\mathrm{th}}$	$2.0\pm1.5_{\mathrm{th}}$	$0.2\pm0.2_{\mathrm{th}}$	$1.1\pm0.3_{\mathrm{th}}$

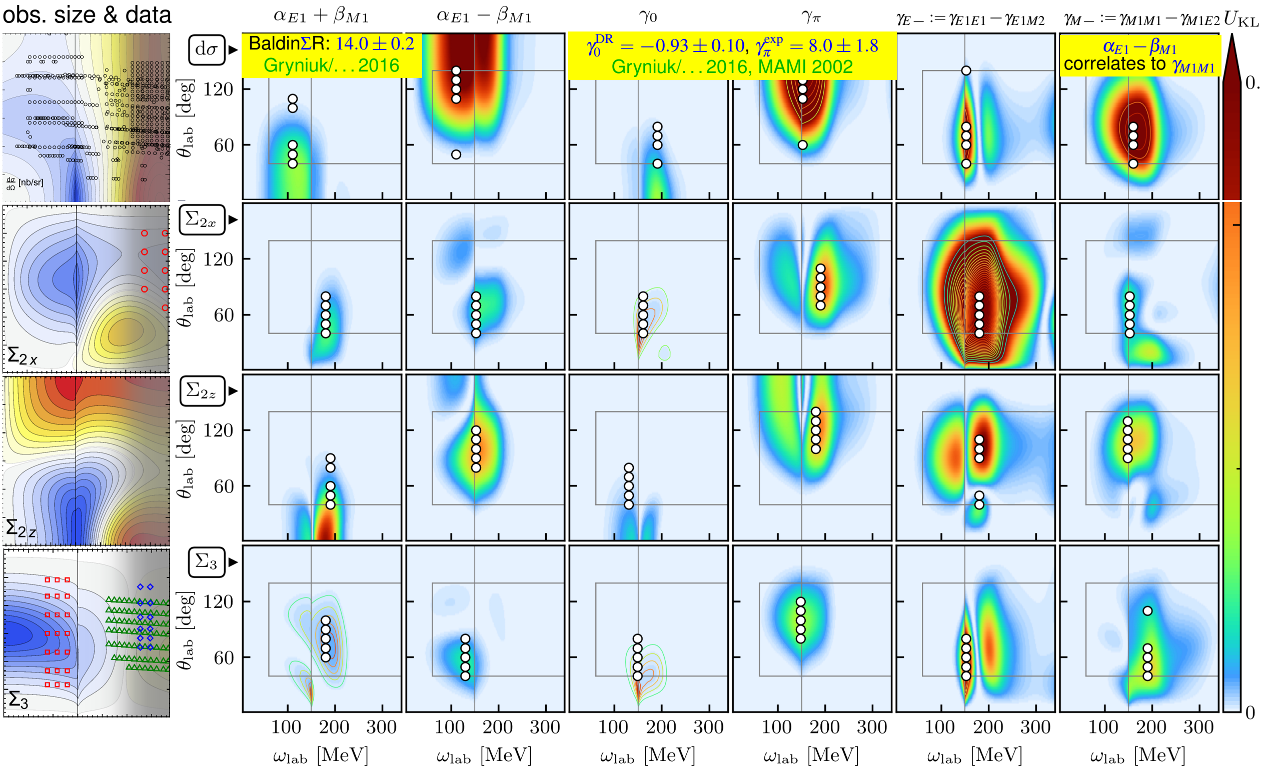
Account for Limitations: Theory most accurate at $\omega \lesssim 230$ MeV. $\Longleftrightarrow$ Experiment high count rates at high ω.

(d) Bayesian Posterior Shrinkage by Intelligent Design

BUQEYE: Melendez/Furnstahl/Pratola/DRP/hgrie/JAMcG/... EPJ **A57** (2021) 81 [2004.11307]

Jupyter notebook at `buqeye.github.io`

OPTIMAL IMPACT MACHINE Proton: Which 5 future angles have biggest impact on a particular polarisability?



$\Longrightarrow$ **Focus on** $d\sigma(100\text{ MeV})$: $\alpha_{E1}-\beta_{M1}$, $d\sigma(160\text{ MeV})$: γ_{M-}; $\Sigma_{2x}(170\text{ MeV})$: γ_{E-} – ***not*** **beam asym.** Σ_3

(e) Experiment and Theory in Sync at the Precision and Intensity Frontier

No single measurement will provide definitive answer: multi-parameter extraction, systematics, validation.

$\Longrightarrow$ **Experiment & Theory collaborate to identify *observables with biggest impact.***

$\Longrightarrow$ ***Mathematica notebooks* on all observables for p, n, deuteron, ^{3}He, ^{4}He available.**

Photon energy ω=120MeV — Reference frame: cm | lab

Deuteron vector polarisation $P_1^{(d)}$=1.1\`

Deuteron tensor polarisation $P_2^{(d)}$=0.53\`

Photon right–circular polarisation $P_{circ}^{(\gamma)}$=–0.5

Photon linear polarisation $P_{lin}^{(\gamma)}$=1.\`

Configuration 1

Deuteron polarisation quantisation axis θ_{d1}=0°

ϕ_{d1}=0°

Photon linear polarisation angle ϕ_{lin1}=90°

Configuration 2

Deuteron polarisation quantisation axis θ_{d2}=90°

ϕ_{d2}=270°

Photon linear polarisation angle ϕ_{lin2}=90°

Variation by ±2 of: $\delta\beta_{M1}$

χEFT order: $e^2\delta^3=\epsilon^3$: with $\Delta(1232)$ | $e^2\delta^2=Q^3$: no $\Delta(1232)$

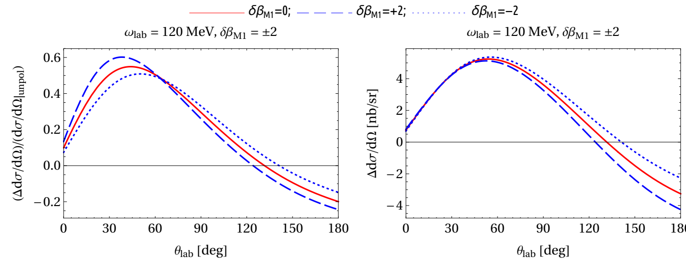
Deuteron wave function: NNLO Epelbaum 650MeV | AV18 — NN potential: AV18

Range on y–axis: All

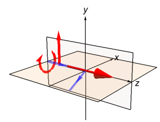
Normalise left plot to: $\frac{d\sigma}{d\Omega}|_{unpol}$ | $\Sigma\frac{d\sigma}{d\Omega}$: sum of configurations

Export $\Delta\frac{d\sigma}{d\Omega}$ and $\Sigma\frac{d\sigma}{d\Omega}$ of this configuration? — File name: "out.dat"

Example double-polarised on deuteron

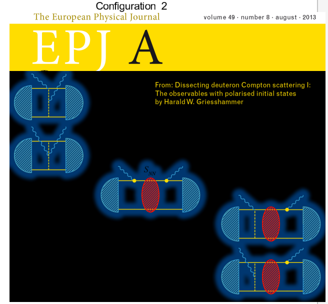


Configuration 1 — Configuration 2



Probability of spin projection $M_{\vec{d}}$:

Cartesian polarisation along $\vec{d}$:



hg EPJA**49** (2013) 100

Europhysics News HIGHLIGHT May 2013

errata A**53** (2017) 113, A**54** (2018) 57

Cross section difference of configurations: $\Delta\frac{d\sigma}{d\Omega}=\frac{d\sigma}{d\Omega}|_{unpol}\times[0+0.78\,T^{lin}_{1,-1}-0.55\,T^{circ}_{1,0}-0.78\,T_{1,1}+0.78\,T^{lin}_{1,1}-0.32\,T^{lin}_{2,-2}+0.8\,T_{2,0}-0.8\,T^{lin}_{2,0}+0.32\,T_{2,2}-0.32\,T^{lin}_{2,2}]$

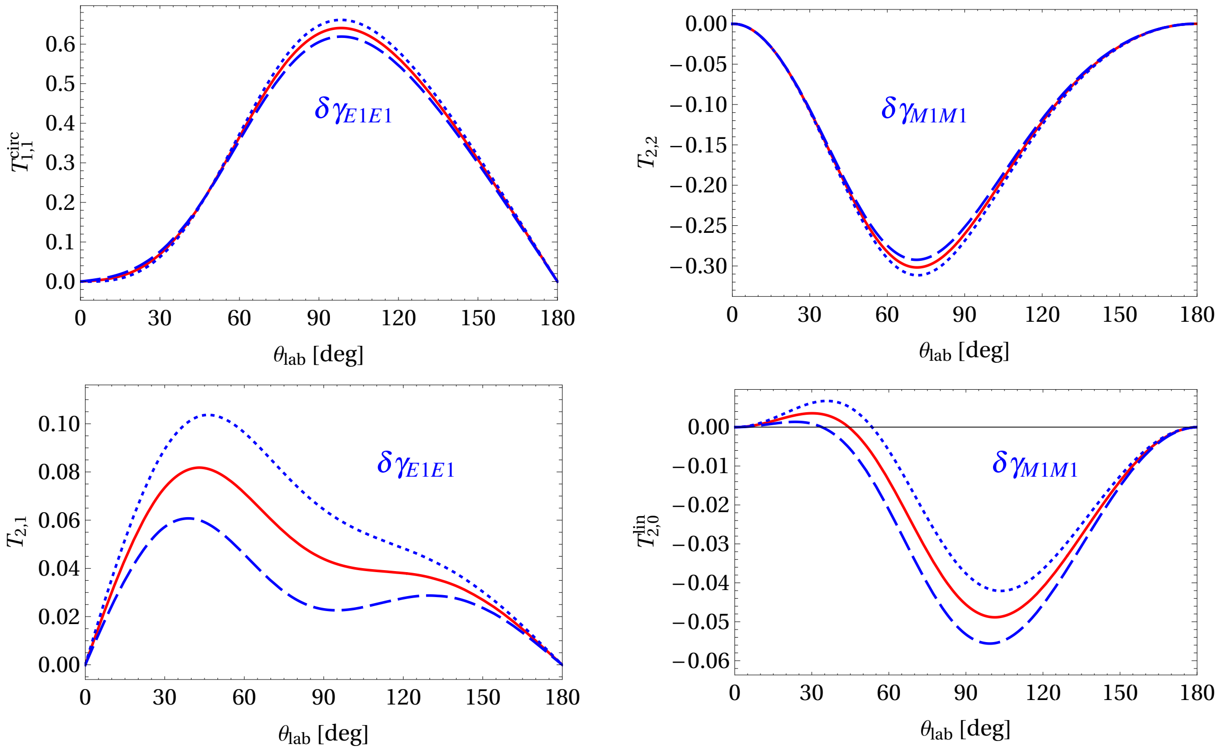
Cross section sum of configurations: $\Sigma\frac{d\sigma}{d\Omega}=\frac{d\sigma}{d\Omega}|_{unpol}\times[2+-2.\,\Sigma^{lin}+-0.78\,T^{lin}_{1,-1}-0.55\,T^{circ}_{1,0}+0.78\,T_{1,1}-0.78\,T^{lin}_{1,1}+0.32\,T^{lin}_{2,-2}+0.26\,T_{2,0}-0.26\,T^{lin}_{2,0}-0.32\,T_{2,2}+0.32\,T^{lin}_{2,2}]$

(f) Vector & Tensor (?!) Deuteron: Spin Polarisabilities at 100 MeV

hg EPJA**49** (2013) 100
err. A**53** (2017) 113
A**54** (2018) 57

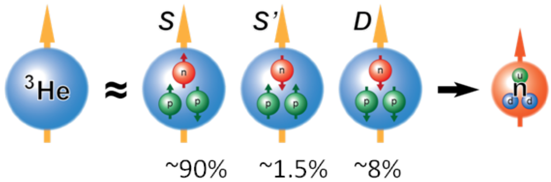
Want very clean observables: Large rates, insensitive to other pols, deuteron wave fu,. . .

central; $\delta\gamma_i = +2$; $\delta\gamma_i = -2$



(g) ^{3}He Double-Polarised: Effective n Spin Target

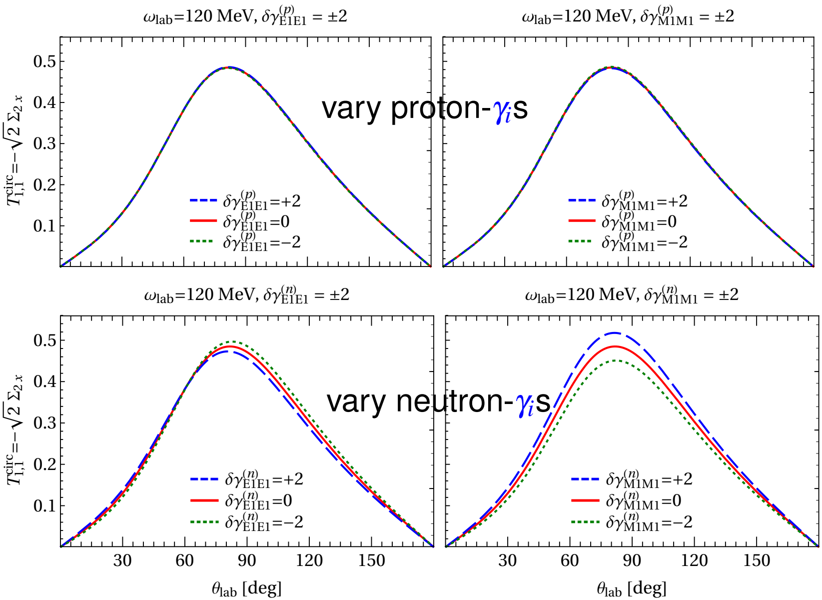
^{3}He as "effective" spin target: *sensitivity* to neutron spin, not to proton spin $(20:1)$.

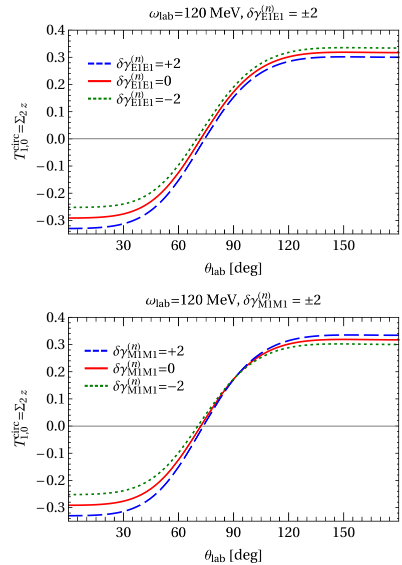


Sensitivity to γ_i's

at $\omega_{lab} = 120$ MeV enhanced by

interference with charged Born+MEC.





4. Concluding Questions

Polarisabilities: **χiral symmetry of pion-cloud, $\Delta(1232)$ properties. Impact on $M_p - M_n$;...**

Spin Polarisabilities: **Stiffness of Spin Constituents; Nuclear Faraday Effect.**

χEFT: systematic, predict & extract with reproducible uncertainties. – LatticeQCD catching up.

Transition Densities for $A \geq 4$: efficient recycling; download package at `pypi.org/project/nucdens/`

Target	Opportunities	Theory Status for All Observables
proton & neutron	nucleon spin polarisabilities	"done": N^{4}LO $\omega \lesssim 230$ MeV for pols. math.nb jupyter.py
deuteron	sensitive to $p+n$ average polarised, d-wave interference: mixed spin pols $\gamma_{E1M2}, \gamma_{M1E2}$	$\omega < m_\pi$ N^{3}LO done, N^{4}LO this year math.nb $\omega \gtrsim m_\pi$ needs resources
^{3}He: increase rates	unpolarised: sensitive to $2p+n$ polarised: "n-spin" $\Longrightarrow \gamma_i^n$ only	densities method [2005.12207] math.nb
^{4}He,^{6}Li: increase rates	sensitive to $p+n$ avg. (^{4}He: not γ_i) test: large $\pi^\pm$ in NN exchange	$\omega \in [50\text{ MeV}; m_\pi]$ N^{3}LO✓, N^{4}LO like d $\omega \to 0$ under way — $\omega \gtrsim m_\pi$ needs resources

We Need Data: **elastic & inelastic cross-sections & asymmetries –** ***reliable systematics!***

Low-ω for scalar, high-ω for spin-polarisabilities, **but always** $\omega \lesssim 230$ MeV.

Only *combination of dedicated experiments* meaningful! (Not "one datum for one answer".)

$\Longrightarrow$ ***Synergy of Experiment, Low-Energy Theory & Lattice QCD, competitive uncertainties!***

$\Longrightarrow$ **Compton Community programme outlined in White Paper for a**
Next Generation Laser Compton Gamma-ray Beam Facility [2012.10843] **and DOE.**

The efficient person gets the job done right. The effective person gets the right job done.

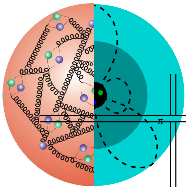
(a) Few-Nucleon Interactions in χEFT

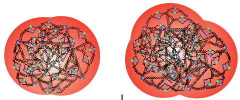
Weinberg, Ordóñez/Ray/van Kolck, Friar/Coon, Kaiser/Brockmann/Weise, Epelbaum/Glöckle/Meißner, Entem/Machleidt, Kaiser, Higa/Robilotta, Epelbaum, ...

$\frac{\text{typ. momentum}}{\text{breakdown scale}} \ll 1$

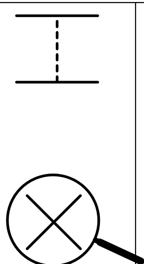
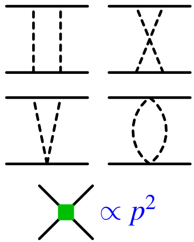
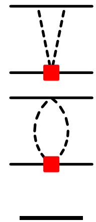
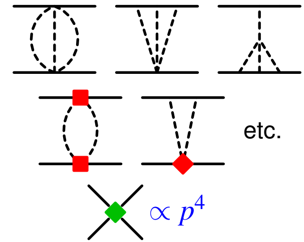
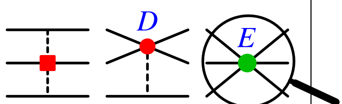
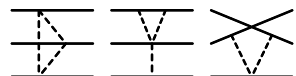
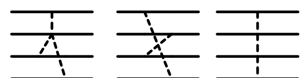
Long-Range: correct symmetries and IR degrees of freedom: **Chiral Dynamics**

Short-Range: symmetries constrain contact-ints to simplify UV: **Minimal parameter-set**





Hierarchy: 2NI-effects $\gg$ 3NI-effects $\gg$ 4NI-effects

	LO	NLO	N^{2}LO	N^{3}LO
$2N$ ints	 2 parameter	 $\propto p^2$ $+7$ parameter	 $+0$ parameter	 etc. $\propto p^4$ $+15 = 24$ param.
$\frac{\chi^2}{\text{d.o.f}}$ in np		36.2	10.1	1.10 (AV 18: 1.04, 40 param.)
$3N$ ints	—	—	 2 parameter	 etc. parameter-free, in progress
$4N$ ints	—	—	—	 etc. parameter-free

(b) Statistical Interpretation of the Max-Criterion: A Simple Example

I take this table of πN scattering parameters in χEFT with effective $\Delta(1232)$ degrees of freedom from a talk by Jacobo Ruiz de Elvira. Here, I am not interested in the Physics, but use it as series $c_i = c_{i0} + c_{i1}\epsilon^1 + c_{i2}\epsilon^2$ in a small expansion parameter.

parameter $[\mathrm{GeV}^{-1}]$	LO total	NLO total	N^{2}LO total	expansion $= c_{i0} + c_{i1}\epsilon^1 + c_{i2}\epsilon^2$	perturbative expansion $\epsilon \approx 0.4$ (guess)
c_1	-0.69	-1.24	-1.11	$= -0.69 + 0.55 - 0.13$	$= -0.69 + 1.38\epsilon^1 - 0.81\epsilon^2$
c_2	$+0.81$	$+1.13$	$+1.28$	$= +0.81 - 0.32 - 0.15$	$= +0.81 - 0.80\epsilon^1 - 0.94\epsilon^2$
c_3	-0.45	-2.75	-2.04	$= -0.45 + 2.30 - 0.71$	$= -0.45 + 5.75\epsilon^1 - 4.44\epsilon^2$
c_4	$+0.64$	$+1.58$	$+2.07$	$= +0.64 - 0.94 - 0.49$	$= +0.64 - 2.35\epsilon^1 - 3.06\epsilon^2$

Now pick the largest absolute coefficient to estimate typical size of next-order correction $c_{i(n+1)} = c_{i3}$ in our case:

Max-Criterion: $c_{i(n+1)} \lesssim \max\limits_{n\in\{0;1;2\}} \{|c_{in}|\} =: R$ is labelled as red in the table.

Multiply that number with ϵ^3 to finally get a corridor of uncertainty/typical size of the ϵ^3 contribution.

For c_1: $\max\limits_{n\in\{0;1;2\}} \{|-0.69|;|1.38|;|-0.81|\} = 1.38 \implies$ error $\pm 1.38 \times (\epsilon = 0.4)^3 \approx 0.09 \implies c_1 = -0.69 \pm 0.09$.

Similar: $c_2 = 1.28 \pm 0.06$, $c_3 = -2.04 \pm 0.37$, $c_4 = 2.07 \pm 0.20$ (round significant figures conservatively).

But what's the statistical interpretation? $\implies$ Next slide!

Notes: **(1)** Provide a theoretical error *estimate* that is *reproducible*. You can then discuss with others who have different opinions. No estimate, no discussion possible. – **(2)** Sometimes, one discards the LO$\rightarrow$NLO correction if it's anomalously large. That is a "prior information" you need to disclose as "bias" of your estimate. – **(3)** Coefficients c_{in} appear "more natural" for c_1 and c_2 than for c_4 – c_4 not that well-converging? – **(4)** The uncertainty estimate is agnostic about the Physics details. Somebody just handed me a table. – **(5)** If you are not happy with the input "$\epsilon \approx 0.4$", pick another number. BUQEYE [1511.03618] developed the Bayesian technology to extract degrees of belief on what value of the expansion parameter the series suggests. – **(6)** The c_i are not observables, but they are renormalised couplings which – according to Renormalisation – should follow a perturbative expansion.

(b) Statistical Interpretation of the Max-Criterion: A Simple Example

The Bayesian interpretation of the max-criterion on the preceding slide provides probability distribution/degree-of-belief (pdf/DoB) functions using a "reasonable" set of assumptions ("priors") which give nice, analytic expressions. That's one choice of assumptions, but other reasonable assumptions provide very similar pdf's see BUQEYE: [1506.01343], [1511.03618],. . . .

But before that, let's do something intuitive which gives the same statistical likeliness interpretation of the max-criterion as the Bayesian one. The Bayesian analysis formalises the example and provides actual pdf's.

Estimating a Largest Number: Given a finite set of (finite, positive) numbers in an urn. You get to draw one number at a time.

Your mission, should you choose to accept it: Guess the largest number in the urn from a limited number of drawings.

For c_1, we first draw $c_{10} = 0.69$. I would say it's "natural" to guess that there is a 1-in-$2 = 50\%$ chance that the next number is lower. But there is also a pretty good chance that if it is higher, then its distribution up there is not Gauß'ian but with a stronger tail.

Next, we draw $c_{11} = 1.38$ which is larger. So I revise my largest-number projection to $R = 1.38$, but I also get more confident that this may be pretty high (if not he highest already). After all, I already found one number which is lower, namely $c_{10} = 0.69$. With 2 pieces of information (0.69 and 1.38), it's "natural" that the 3rd drawing has a 2-in-3 or $2/3$ chance to be lower.

Next, we draw $c_{12} = 0.81 < R$. Looking at my set of 3 numbers, I am even more confident that $R = c_{11} = 1.38$ is the largest number, with 3-in-4 or 75% confidence. For c_1, evil forces interfere and we have no more drawings to draw information from.

But if we could reach into the urn k times and look at the collected k results, every time revising our max-estimate, it's "natural" to assign a $100\% \times k/(k+1)$ confidence that I have actually gotten the largest number R.

The Bayesian procedure on the next slide provides the same result. Read the BUQEYE papers for details and formulae!

In our example, we had $k = 3$ terms (drawings) for c_1. So the confidence that $R = 1.38$ is indeed the highest number is $3/4 = 75\%$, which is larger than $p(1\sigma) \approx 68\%$. For a 1σ corridor, I reasonably assume that the numbers are equi-distributed between 0 and the maximum R. Then, the 68%-error corridor is set by $\pm 68\% \times (k+1)/k \times R$ amongst the known numbers.

Now, I multiply that number with 3 powers of the expansion parameter $\epsilon \approx 0.4$ (estimate N^{3}LO terms!) (but see **Note (5)** on the previous slide): $\pm 1.38 \times (68\%/75\%) \times 0.4^3 = \pm 0.08$ is a good uncertainty estimate for a traditional 68% confidence region. I also get a feeling that the probabilities outside the interval $[0;R]$ may not be Gauß'ian-distributed. Bayes confirms that.

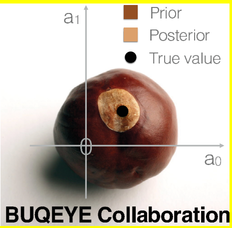
(c) Theorists Have Error Bars: "Truncation" Errors!

χEFT $\alpha_{E1}^{(p)} - \beta_{M1}^{(p)}\ [10^{-4}\ \text{fm}^3]: 7.5 \pm ???_{\text{th}} = 11.2_{\text{LO}} - 3.6_{\text{NLO}} - 0.1_{\text{N}^2\text{LO}} \pm ???_{\text{th}}$

Observable as series of k terms to N^{k-1}LO: $\mathcal{O}^{(k=2)} = c_0 + c_1\,\delta^1 + c_2\,\delta^2$ +unknown $c_3 \times \delta^3$

Assuming $\delta \simeq 0.4$: $11.2 \quad -9.1\,\delta^1 \; -0.6\,\delta^2$ +unknown $\times\,\delta^3$

$\Longrightarrow$ **Estimate next term *"most conservatively"* as** $|$unknown $c_3| \lesssim c_{\max} := \max\{|c_0|;|c_1|;|c_2|\}$.



No infinite sampling pool; data fixed; more data changes confidence.

Call upon the Reverend Bayes for probabilistic interpretation!

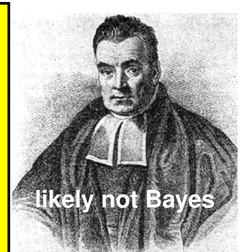
e.g. **BUQEYE collaboration**

Furnstahl/Phillips/... [1506.01343]+[1511.01952]+...

New information increases level of confidence.

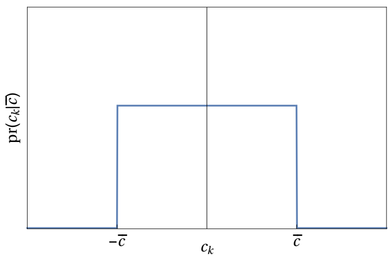
$\Longrightarrow$ **Smaller corrections, more reliable uncertainties.**

Clearly state your premises/assumptions – including *naturalness.*



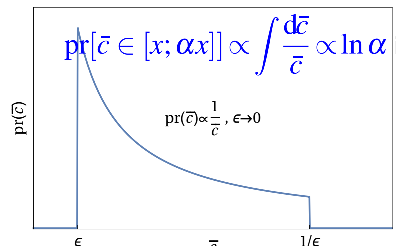
Priors: leading-omitted term dominates ($\delta \ll 1$); putative distributions of *all* c_k's and of largest value $\bar{c}$ in series.

Uniform ~~"least informed/-ative"~~: All values c_k equally likely, given upper bound $\bar{c}$ of series.



"Any upper bound" (*Benford's Law*): $\ln$-uniform prior sets no bias on scale of $\bar{c}$

$\text{pr}[\bar{c} \in [x;\alpha x]] \propto \int \frac{\mathrm{d}\bar{c}}{\bar{c}} \propto \ln\alpha$ indep. of x



equi-distributed

(c) Theorists Have Error Bars: "Truncation" Errors!

max-criterion: lore since "time immemorial"
Bayes: e.g. Cacciari/Houdeau [1105.5152]
BUQEYE [1506.01343]+[1511.03618]
applied in hg/JMcG/DRP [1511.01952]

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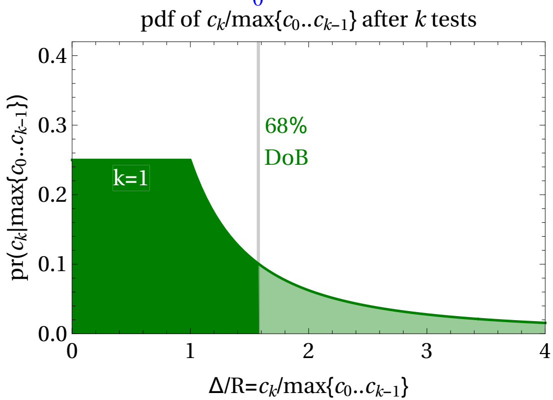
$$\mathrm{pr}(x|c_{\max},\mathrm{order}\ k) \propto \int\limits_0^\infty \mathrm{d}\bar{c}\ \mathrm{pr}(\bar{c})\ \mathrm{pr}(x|\bar{c}) \prod_{n=0}^{k-1}\mathrm{pr}(c_n|\bar{c}) \to \frac{k}{k+1}\,\frac{1}{c_{\max}}\begin{cases}1 & x\le c_{\max}\\ \dfrac{1}{x^{k+1}} & x> c_{\max}\end{cases}$$

BUQEYE [1506.01343] eq. (22)



Priors: all c_n "equally likely", "any upper bound" $\bar{c}$.

order	in $\pm c_{\max}$	$\Delta^{(k)}(68\%)$	$\Delta^{(k)}(95\%)$
LO	$\frac{1}{2}=50\%$	$1.6\ c_{\max}$	$11 c_{\max} = 7\ \Delta_{68}^{(1)}$
Gauß	68.27%	$1.0\ c_{\max}$	$2.0\ \Delta_{68}^{(k)}$

Laplace's Law of Succession (flat prior, T/F): Chance that next coefficient is $< c_{\max}$: $\dfrac{k}{k+1}$.

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Assuming $\delta \simeq 0.4$: $11.2\ \mathbf{-9.1}\ \delta^1 - 0.6\,\delta^2$ +unknown $\times\ \delta^3$

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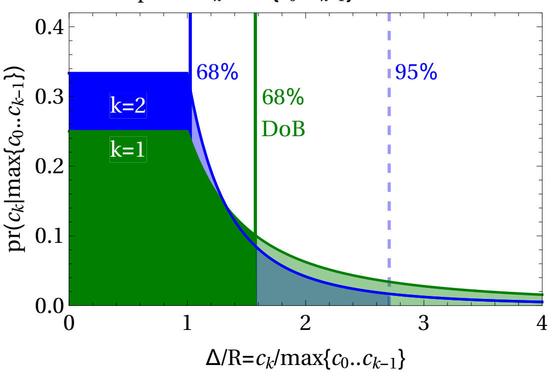

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Assuming $\delta \simeq 0.4$: $11.2 \quad -9.1\,\delta^1 \; \mathbf{-0.6}\,\delta^2 \quad \pm(11.2 \times \delta^3 \approx 0.7??)$

$\Longrightarrow$ **Estimate next term *"most conservatively"* as** $|$unknown $c_3| \lesssim c_{\max} := \max\{|c_0|;|c_1|;|c_2|\}$.

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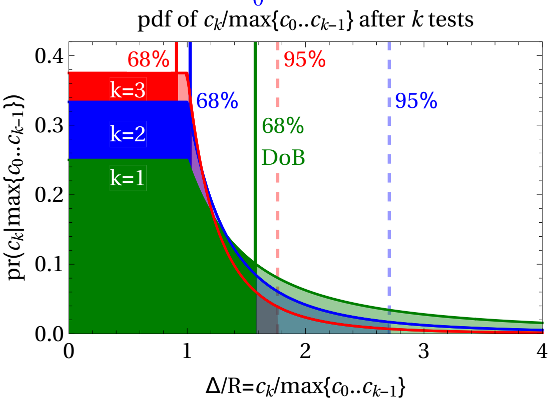

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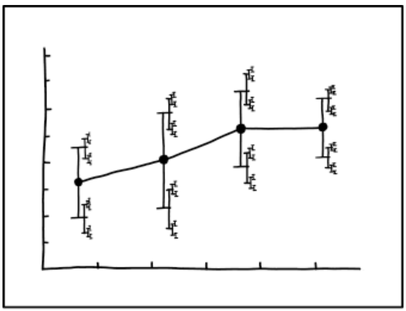
NLO	$\frac{2}{3} = 66.7\%$	$1.0\ c_{\max}$	$2.7 c_{\max} = 2.6\ \Delta_{68}^{(2)}$
N^{2}LO	$\frac{3}{4} = 75\%$	$0.9\ c_{\max}$	$1.8 c_{\max} = 1.9\ \Delta_{68}^{(3)}$
N^{k-1}LO k terms	$\frac{k}{k+1}$	$0.68\frac{k+1}{k} c_{\max}\ (k \ge 2)$	
Gauß	68.27%	$1.0\ c_{\max}$	$2.0\ \Delta_{68}^{(k)}$

$\Longrightarrow$ **Use theory uncertainties with these priors: "$\mathcal{O}^{(k)} \pm \Delta_{68}^{(k)}$": 68% *DoB interval* $[\mathcal{O}^{(k)} - \Delta_{68}^{(k)}; \mathcal{O}^{(k)} + \Delta_{68}]$.**

(d) How To Spend Your Time & Money Wisely!

Deliberate experimental planning needs to integrate
theory $\oplus$ experimental facts $\oplus$ likeliness of success
to optimise money & time & workforce & reputation in suite of future measurements!
$\Longrightarrow$ Gain knowledge, test theories, find new effects.

Outcome: **"Optimal Impact Machine" (generally accepted/well-defined/reproducible/canned) to identify sequence of experiments with likely high(est) impact & chance of success: strategically placed data with excellent Figures of Merit for more-accurate theory validation, parameter extractions,...**

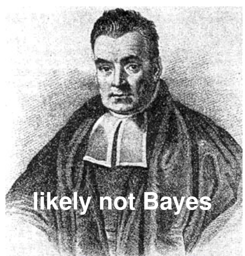


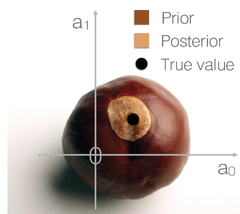
I DON'T KNOW HOW TO PROPAGATE ERROR CORRECTLY, SO I JUST PUT ERROR BARS ON ALL MY ERROR BARS.
xkcd 13.02.2019

Bayesian Analysis: **Be up-front about assumptions.**
Reasonable people can reasonably disagree about reasonable assumptions, but no reasonable discussion without specifying them.

Test stability under reasonable variations of assumptions.

`buqeye.github.io`: ***Jupyter notebooks*** **apply beyond Compton.**





BUQEYE Collaboration

(e) Bayesian Posterior Shrinkage by Intelligent Design

OPTIMAL IMPACT MACHINE: Maximise benefits – minimise cost (time, money, workforce, data not taken).

Input: (1) **Present polarisability errors** $\Delta\alpha\beta\gamma$ (th & exp, some correlated) – values $\alpha\beta\gamma$ irrelevant.

(2) χ**EFT Predictions** with truncation errors via $\mathcal{GP}$, increasing as $\omega\nearrow$. posterior predictive distr.

(3) **New Data Position** $\overrightarrow{\omega\theta}$: We took 1 energy with 5 angles (exp. constraints) – values $\boldsymbol{y}(\overrightarrow{\omega\theta})$ irrelevant.

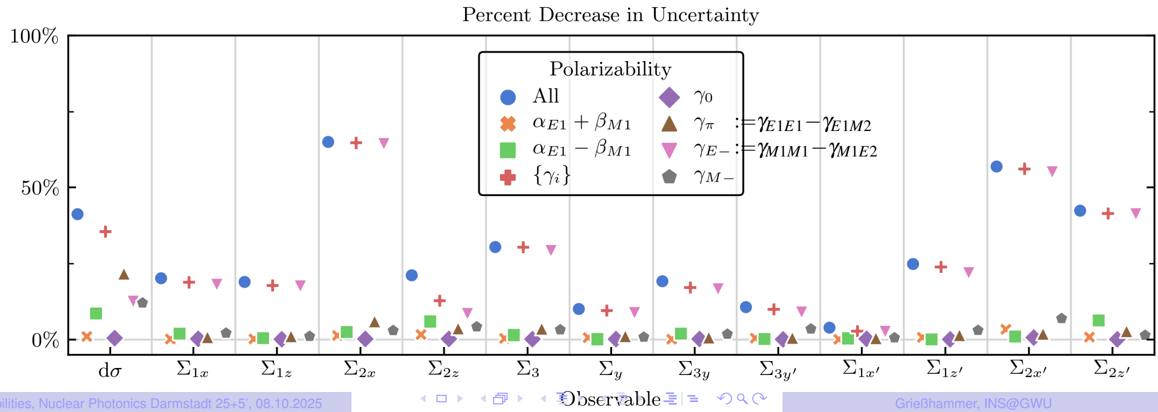
(4) **New Data Quality**: **"Doable ($)"**: cross sections to $\pm 4\%$, asymmetries to ± 0.06 (absolute).

(3+4) $=$ **Expert Elicitation:** Could also add direct penalties for cost, beamtime, event rate,…

here pragmatic: impact of existing data via fits of $\alpha\beta\gamma$; choose uniform exp. constraints.

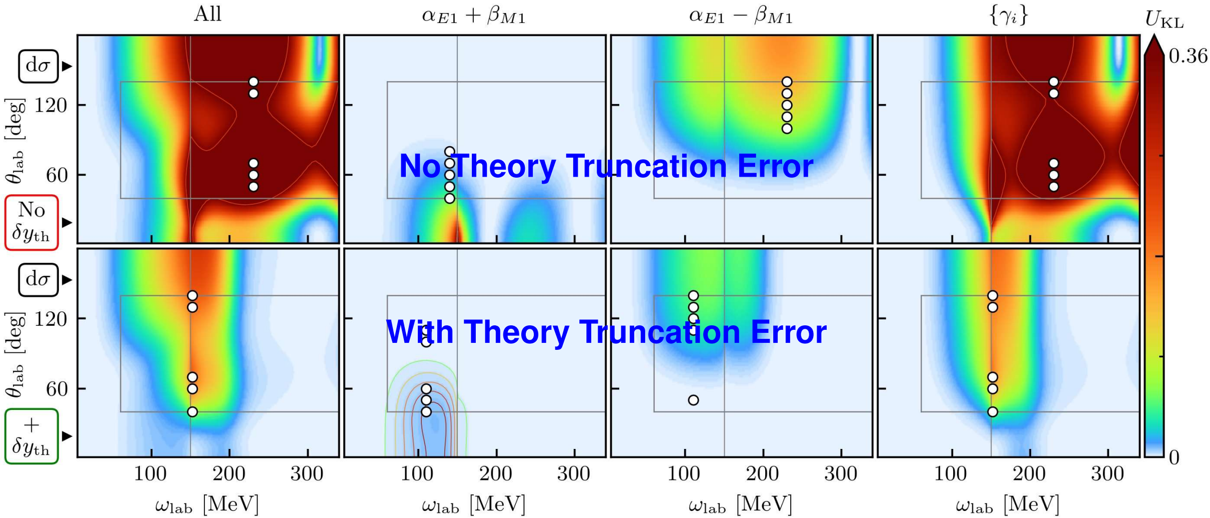
Utility Gain: What new data at points $\overrightarrow{\omega\theta}$ with results $\boldsymbol{y}$ guessed from theory (with errors) gives likely biggest

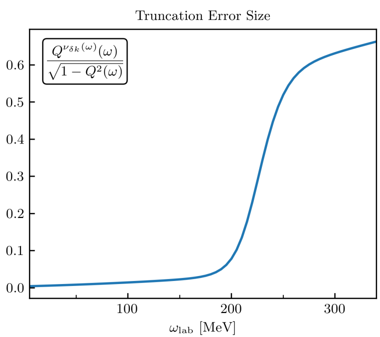
$$U_{\mathrm{KL}} = \underbrace{\int \mathrm{d}\boldsymbol{y}\ \mathrm{pr}(\boldsymbol{y}|\overrightarrow{\omega\theta}) \int \mathrm{d}(\alpha\beta\gamma)}_{\text{data } \boldsymbol{y} \text{ \& central } \alpha\beta\gamma \text{ marginalised}}\ \underbrace{\mathrm{pr}(\alpha\beta\gamma|\boldsymbol{y},\overrightarrow{\omega\theta}) \ln\frac{\mathrm{pr}(\alpha\beta\gamma|\boldsymbol{y},\overrightarrow{\omega\theta})}{\mathrm{pr}(\alpha\beta\gamma)}}_{\text{Shannon information gain}} \underset{\text{linearisation works very well}}{\approx} \ln\left\langle\frac{\text{error's hypervolume before}}{\text{error's hypervolume after data}}\right\rangle_{\mathrm{avg}}$$



(e) Bayesian Posterior Shrinkage by Intelligent Design

BUQEYE: Melendez/Furnstahl/Pratola/DRP/hgrie/
JAMcG/... EPJ**A57** (2021) 81 [2004.11307]
Jupyter notebook at `buqeye.github.io`





$$\mathcal{O} = c_0 + c_2\delta^2 + c_4\delta^3 + c_4\delta^4 + \dots$$

$$\delta = \sqrt{\frac{p_{\mathrm{typ}} \sim (\omega \sim m_\pi \nearrow \Delta_M)}{\overline{\Lambda}_\chi}}$$

Forgetting EFT Truncation Error

Over-Estimates Signal (scale changed!)

Over-Emphasises Resonance Region!

$\Longrightarrow$ **Wrong data point decision!**